

## DOCUMENT RESUME

ED 115 490

SE 019 853

AUTHOR Romberg, Thomas A., Ed.  
TITLE Four Studies on Aspects of Assessing Computational Performance. Technical Report No. 297.  
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.  
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.  
REPORT NO WRDCCL-TR-297  
PUB DATE Jul 75  
CONTRACT NE-C-00-3-0065  
NOTE 115p.; Report from the Project on Conditions of School Learning and Instructional Strategies

EDRS PRICE MF-\$0.76 HC-\$5.70 Plus Postage  
DESCRIPTORS Addition; \*Basic Skills; Elementary Education; \*Elementary School Mathematics; Instruction; Mathematics Education; \*Research; Subtraction; \*Test Construction; \*Testing  
IDENTIFIERS Research Reports

## ABSTRACT

The four studies reported in this document deal with aspects of assessing students' performance on computational skills. The first study grew out of a need for an instrument to measure students' speed at recalling addition facts. This had seemed to be a very easy task, but it proved to be much more difficult than anticipated. The second study grew out of a need to develop a test of addition and subtraction which was both machine scorable and not multiple choice. The study reported here compares a new response format to two other response formats. The third study deals with the question of item ordering on tests. The study concerns tests with items ordered from easiest to hardest, hardest to easiest, and randomly, and how item ordering interacts with student anxiety levels. The fourth study deals with three remedial methods of instruction for students at the second-grade level who were unable to perform two-digit addition with regrouping. One method used the mini-calculator as an aid, the second used a partial sums approach, and the third the traditional regrouping method. (Author/SD)

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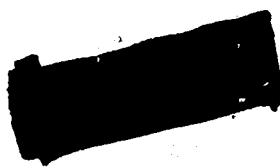
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TECHNICAL REPORT NO. 297

# four studies on aspects of assessing computational performance



JULY 1975

WISCONSIN RESEARCH  
AND DEVELOPMENT  
CENTER FOR  
COGNITIVE LEARNING



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Technical Report No. 297

FOUR STUDIES ON ASPECTS  
OF ASSESSING COMPUTATIONAL PERFORMANCE

Edited by

Thomas A. Romberg

Report from the Project on  
Conditions of School Learning and Instructional Strategies

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July 1975

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the National Institute of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.

Center Contract No. NE-C-00-3-0065

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- developing improved instructional strategies, processes and materials for school administrators, teachers, and children, and
- offering assistance to educators and citizens which will help transfer the outcomes of research and development into practice

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The activities of the Wisconsin R&D Center are organized around one unifying theme, Individually Guided Education.

### FUNDING

The Wisconsin R&D Center is supported with funds from the National Institute of Education; the Bureau of Education for the Handicapped, U.S. Office of Education; and the University of Wisconsin.

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## ABSTRACT

This report documents four brief studies. Each deals with an aspect of assessing pupil performance on computational skills.

### Study 1: Preliminary Development of an Oral Speed Test for Arithmetic Facts

Speed of recalling computational facts has long been considered a prerequisite to learning algorithms. However, research does not exist to confirm or deny this contention. There is not even a reliable procedure for ascertaining speed of factual recall. This study reports initial efforts to develop an oral speed test for addition facts. As a result, a reasonable instrument to measure speed of factual recall appears to be possible.

### Study 2: An Investigation of Three Different Response Formats for a Two-Digit Addition and Subtraction Test

This study was designed to see if responses to an addition and subtraction test administered via a new machine-scorable completion format (shade the circle) were as reliable as responses over a simple completion format or a multiple-choice format. The results suggest that the new format is a viable alternative for tests of computational algorithms.

### Study 3: The Effect of Test Item Ordering on the Performance of Children of Different Test Anxiety Levels

The ordering of items and the effect on performance of children with different test anxiety levels was examined in this study. Three tests were constructed from the same set of items: Test A had items ordered from easiest to hardest, Test D from hardest to easiest, and Test M had items randomly ordered. Three test anxiety groups were determined using the test anxiety scale for children. Results failed to show significant differences in performance for either item ordering or for the anxiety level. However, somewhat higher means were consistent for the mixed order and the low anxiety groups.

### Study 4: A Comparative Study of Three Remedial Methods of Instruction for Two-Digit Addition with Regrouping at the Second Grade

In this experimental study a group of second-grade students needing remedial instruction on the two-digit addition algorithm were randomly

assigned to three treatment groups: one utilizing the hand-held calculator, one using a sub-sums approach, and one using the traditional approach. All three methods produced increased achievement. The calculator method, however, yielded the poorest results; the traditional carrying method yielded the best.

## INTRODUCTION TO THE FOUR STUDIES

The four studies reported in this document deal with some aspect of assessing students' performance on computational skills. The studies are part of an ongoing program of research under the direction of Professors Thomas A. Romberg and J. Fred Weaver at the University of Wisconsin. The program gained impetus in 1973 when Professor Weaver directed a presession on algorithmic learning for the AERA Special Interest Group in Research in Mathematics Education.

The first study, titled Preliminary Development of an Oral Speed Test for Arithmetic Facts, grew out of a need for an instrument to measure students' speed at recalling addition facts. This had seemed to be a very easy task, but it proved to be much more difficult than anticipated.

The second study grew out of a need to develop a test which was both machine scorable and not multiple choice and which was designed to ascertain students' ability to add and subtract. The study reported here compares a new response format to two other response formats.

The third study deals with the question of item ordering on tests. The question was raised as to whether items should be ordered from easiest to hardest, hardest to easiest, or randomly. The question was posed in terms of student anxiety levels.

The fourth study deals with three remedial methods of instruction for students at the second-grade level who were unable to perform two-digit addition with regrouping. One method used the mini-calculator as an aid, the second used a partial sums approach, and the third the traditional regrouping method.

1/2

STUDY I

PRELIMINARY DEVELOPMENT OF AN  
ORAL SPEED TEST FOR ARITHMETIC FACTS

INTRODUCTION

This report describes initial attempts to develop a valid and reliable measure of students' speed at recalling addition facts. This work is seen as a first step in a sequence of investigations into algorithmic learning. In particular, the material developed here will be used to examine the relationship between students' rate of response on basic arithmetic combinations (facts) and their learning of algorithms that use those basic combinations.

Research has little to say concerning the relationship of students' speed on facts to their learning of algorithms. However, it is generally believed by teachers that rapid response rates by students on the basic facts are prerequisite to their learning the algorithms that employ the facts. The proliferation of "drill and practice" programs for use in the elementary school supports the contention that such response rates are highly valued instructional outcomes. The immediate concern of the Wisconsin Research and Development Center is the need to specify guidelines for the development of a "drill and practice" program for Developing Mathematical Processes (1973).

Two studies related to the general problem are reported by Brownell (1953) and Wiles (1973). In a study of the effects of learning an algorithm for division by two-place numbers on constituent skills, Brownell reported that practice with the complex skill had no single predictable result as far as proficiency in subskills was concerned. Wiles (1973) studied the effects of sequence of instruction on the acquisition of addition and subtraction algorithms. Measures of pre-instruction and post-instruction rates of response with the basic addition and subtraction facts were included as part of the design. These measures showed no predictable effects on students' rate of responding to the basic facts as a result of six weeks' instruction on the algorithms; in fact, on three of the six measures students showed absolute decreases in their rate of responding. These studies relate to what might be called the "incidental drill" hypothesis. However, the question of the relative importance of students' speed on facts to their learning of algorithms that employ those facts is not answered.

Thus, an investigation into the relationship between students' speed on facts and their learning of algorithms was called for. For this proposed study addition facts and the addition algorithm were chosen because (1) children who do not know an addition algorithm probably do not know any other computational algorithms and (2) instructional packages that enable large proportions of a group of learners to master the addition

algorithm are readily available and it should be possible to find children with a broad ability range who vary widely in their rates of response to the basic facts and who do not know an adequate additional algorithm for two- or three-digit numerals.

### DETAILS OF THIS STUDY

To carry out the proposed investigation, it was necessary to develop a valid and reliable way of measuring students' speed on addition facts. A previous measure (Wiles, 1973) was inadequate in two ways. First, it was a speed test that involved both addition and subtraction facts. Facts of both types were presented so that no more than two items of any type appeared consecutively.

Second, the posttest revealed algorithmic interference characterized by the examples in Figure I.1.

$$\begin{array}{r}
 \begin{array}{r}
 6 \\
 +8 \\
 \hline
 14
 \end{array}
 \quad
 \begin{array}{r}
 01 \\
 \times 1 \\
 \hline
 -9 \\
 02
 \end{array}
 \quad
 \begin{array}{r}
 15 \\
 -6 \\
 \hline
 19
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 +4 \\
 \hline
 10
 \end{array}
 \quad
 15-9 = 6
 \end{array}$$

Figure I.1. Examples of student work illustrating algorithmic interference.

To avoid algorithmic interference it had seemed reasonable that the visual presentation of the facts in the vertical form would encourage students to treat basic facts like a problem that is efficiently solved by an algorithm. However, as is apparent from the last example in Figure I.1, placing the facts item in a horizontal form did not prevent interference from occurring. Thus, it was decided to ask the student to respond to oral stimuli and to record only the sum.

Three different oral tape recordings were made with response booklets for students to record their answers. The first of the three tapes, Tape 1, was piloted with a total of 13 third graders and 10 second graders. Data obtained from the response booklets and the observations of the test administrator were used in designing the script for the second and third tapes, Tape 2A and Tape 2B. Another test, called the "three-form test," was then constructed. Each form of the three-form test presents the same items as Tapes 2A and 2B.

### Specifications of Tape 1

Tape 1 contains 7 sets of 10 addition facts read at 7 various time intervals (see Table I.1). Each set of 10 facts consists of 5 easy items and 5 difficult items. An easy item is defined as one where the indicated sum of 2 one-digit numbers (neither zero) is greater than 2, but less than 10. A difficult item then is defined as one where the indicated sum of 2 one-digit numbers is 10 or greater. Each of the 7 sets of 10 items was randomly generated. Then the items were ordered subject to the following constraints:

1. No more than 2 easy or 2 difficult items occur consecutively.
2. No item occurs more than once in the 7 sets of 10 items.
3. An item and its commuted form are not allowed to occur within the same set of 10 items!
4. Each set of 10 items contains 4 indicated sums of the following characteristics:
  - a. one item whose sum is exactly 10,
  - b. one item that is a double, such as  $4 + 4$ ,
  - c. two items where one of the numbers to be added is 9.

The 7 sets of items were read at time intervals as indicated in Table I.1 and recorded on magnetic tape. The interval was measured from

TABLE I.1  
TIME INTERVALS FOR TAPE 1 ITEM BLOCKS

| Item Block | Time Intervals in Seconds |
|------------|---------------------------|
| 1          | 15                        |
| 2          | 15                        |
| 3          | 10                        |
| 4          | 8                         |
| 5          | 6                         |
| 6          | 4                         |
| 7          | 10                        |

the beginning of the reading of one item to the beginning of the next item (or the command to stop). Then the time required to read the item was included as part of the indicated interval. The script for Tape 1 is contained in Appendix A.

The first block of items was intended simply to familiarize the children with the general format of the test and the task requirements. The last block of items was given at the slow rate of 1 per 10 seconds so that the children would finish the tape with a feeling of success. The second through the sixth blocks were given at progressively shorter intervals to find the speed at which the child could no longer produce correct responses.

The purpose of Tape 1 was to measure the speed at which students can produce correct answers. Hence, a subject must have some means of determining correct sums. The second 15-second interval block was intended as a power test measure of a student's ability to do this. The decreasing time intervals for blocks 3 through 6 were intended to identify differences in children's ability to do this rapidly. Of course, a child who has memorized all the facts should be able to produce answers much more rapidly than a child who must count out sums.

#### The Pilot of Tape 1

Tape 1 was administered to two groups of third graders and one group of second graders in Fall 1972. All of the subjects were from a suburban school in a metropolitan school district in central Wisconsin. In addition to the tape, the children were also asked to work some two-digit addition problems to see if they had mastered an algorithm. The children were allowed all the time they wanted to complete the two-digit items.

One of the third-grade groups was a selected group of 7 children identified by their teacher as of average or less than average mathematical ability. This group is referred to as III-M. The other third-grade group of six children was identified by the same teacher as being of high mathematical ability. This second group is referred to as III-H. The second-grade children, referred to as group II, were a random selection of 10 children from an intact second-grade class.

Table I.2 contains the data obtained from the pilot of Tape 1. The variable ANC measures student ability to find the sum of 2 two-digit numbers that do not require regrouping, and the variable AC measures student ability to complete such two-digit items that do require regrouping. The third-grade children, groups III-H and III-M, were asked to work 10 items of each type. The second-grade children were asked to work only 1 item of each type because they had received no formal instruction with two-digit algorithms. The data indicate that none of the second graders could add using regrouping but that 4 of the 10 could correctly complete an item



TABLE 1.2  
DATA FROM PILOT OF TAPE 1

| Subjects<br>by Group | Numbers of Incorrect Items for Each Variable |    |  |      |      |     |     |     |      |
|----------------------|--|----|--|------|------|-----|-----|-----|------|
|                      | Two-Digit<br>Algorithms                      |    | Tape Block/Time Interval in Seconds<br>(easy items, difficult items) |      |      |     |     |     |      |
|                      | ANC  | AC | 1/15   | 2/15 | 3/10 | 4/8 | 5/6 | 6/4 | 7/10 |
| III-H                |  |    |  |      |      |     |     |     |      |
| BE                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,0 | 0,0 |      |
| AU                   | 2  | 1  | 0,0  | 0,0  | 0,0  | 0,0 | 0,0 | 0,0 |      |
| E                    | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,1 | 0,0 | 0,0 |      |
| JL                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,1 | 0,1 | --   |
| NM                   | 0  | 8  | 0,0  | 0,0  | 0,0  | 0,0 | 0,2 | 0,1 | --   |
| DS                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,0 | 0,0 | --   |
| III-M                |  |    |  |      |      |     |     |     |      |
| SF                   | 0  | 9  | 0,0  | --   | 0,2  | 0,0 | 0,3 | 0,0 | 0,0  |
| B                    | 0  | 0  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 0,0 | 0,0  |
| Ti                   | 0  | 10 | 0,1  | --   | 0,0  | 0,2 | 0,0 | 0,2 | 0,0  |
| Am                   | 1  | 10 | 0,0  | --   | 0,0  | 0,0 | 1,2 | 0,0 | 0,0  |
| Ot*                  | 8  | 8  | 0,1  | --   | 0,2  | 0,1 | 0,0 | 0,0 | 0,0  |
| SB                   | 0  | 9  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 0,0 | 0,0  |
| Te                   | 0  | 9  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 1,0 | 0,0  |
| II**                 |  |    |  |      |      |     |     |     |      |
| JP                   | No   | No | 0,0  | 0,0  | 0,0  | 0,0 | 0,1 | 0,2 | 0,2  |
| Be                   | Yes  | No | 0,4  | 0,1  | 0,4  | 0,2 | 0,3 | 1,5 | 0,4  |
| Ch                   | Yes  | No | 0,1  | 0,4  | 0,2  | 0,3 | 0,3 | 1,2 | 0,3  |
| Mi                   | No   | No | 1,3  | 1,2  | 2,2  | 0,3 | 1,4 | 0,2 | 1,3  |
| Ro                   | Yes  | No | 0,0  | 0,0  | 0,1  | 0,0 | 0,0 | 1,0 | 0,1  |
| Ji                   | No   | No | 0,0  | 0,0  | 0,2  | 0,0 | 0,3 | 1,2 | 1,1  |
| Gr                   | No   | No | 0,1  | 1,4  | 0,2  | 0,4 | 1,3 | 2,4 | 0,3  |
| Ci                   | No   | No | 0,3  | 2,4  | 0,3  | 0,3 | 3,4 | 1,5 | 2,3  |
| Ke                   | Yes  | No | 2,1  | 0,4  | 1,3  | 1,2 | 2,4 | 1,4 | 0,0  |
| Su                   | No   | No | 1,3  | 3,4  | 1,3  | 1,3 | 1,4 | 4,5 | 4,4  |

\*This subject counted his sums for ANC and AC using tally marks.

\*\*Group II attempted only one item for variables ANC and AC.

"Yes" means it was correct and "No" means it was not.

that did not require regrouping. Only 1 of the third graders in the medium-low ability group knew an adequate algorithm while 5 of the 6 high ability third graders did.

Tape 1 was administered first to group III-H, then to group III-M, and finally to group II. During the administration of the tape to group III-H, it was apparent that they were making very few errors. In fact, it was only during blocks 5 and 6 that they showed any concern at all about the time intervals. The subject SU who missed 2 items for block 3--the 10-second interval block--apparently did so because he was bored with the whole procedure. Rather than risk a complete loss of task focus by returning to the 10-second intervals of block 7, it was decided to not play that block for these children. The performances of these children for the variables ANC and AC indicated that all but one of these children did possess an adequate algorithm for the addition of two-digit whole numbers. All of these children demonstrated mastery with the facts as evidenced by their perfect scores for blocks 1 and 2.

The subjects of group III-M also demonstrated mastery with the facts as demonstrated by the fact that only two of the subjects missed any items on the warm-up block 1, and these children only missed one item each. Block 2 was not played for these children because of the loss of task focus that was observed during the administration of the tape to group III-H. The subjects of group III-M, however, still appeared to waver in their attention during the playing of block 3. One of the seven subjects of group III-M demonstrated knowledge of an adequate addition algorithm; he also responded correctly to all of the items of each block.

In spite of the fact that the third-grade groups III-H and III-M appeared to loose interest in the task during the course of blocks 1, 2, and 3, it was decided that the second graders of group II could be different enough from the third graders to warrant an administration of all seven of the blocks. However, the experimenter noted that during the administration of blocks 1 and 2, the amount of time provided was obviously longer than the children required to determine their responses. It was conjectured that dawdling between items was carried over into block 3, adversely affecting their scores for this block. The children became aware, however, as they worked on block 3, that less time was allowed per item, and they began to focus more sharply on the task. The data from group II indicated that only 4 of the 10 subjects had mastered the ability to produce correct responses to the facts, and that none of the 10 children of group II knew an adequate addition algorithm for two-digit whole numbers.

The subjects for whom this test was intended are children who have mastered the ability to produce correct sums to the basic addition facts, and who have either not mastered an addition algorithm or have only recently done so. A total of 11 subjects satisfied the criterion of mastery of the facts (80 percent correct) and failure to demonstrate knowledge of an adequate addition algorithm. Table I.3 contains the scores of these 11

subjects. Also included in Table I.3 are the data of 2 group II subjects who demonstrated mastery of the facts from Tape 1, block 1, but not from Tape 1, block 2.

TABLE I.3  
SUBJECTS WHO MASTERED FACTS BUT NOT AN ADDITION ALGORITHM

| Subjects<br>by Group | Numbers of Incorrect Items for Each Variable |    |  |      |      |     |     |     |      |
|----------------------|--|----|--|------|------|-----|-----|-----|------|
|                      | Two-Digit Algorithms                         |    | Tape Block/Time Interval in Seconds<br>(easy items, difficult items) |      |      |     |     |     |      |
|                      | ANC  | AC | 1/15   | 2/15 | 3/10 | 4/8 | 5/6 | 6/4 | 7/10 |
| III-H                |  |    |  |      |      |     |     |     |      |
| NM                   | 0  | 8  | 0,0  | 0,0  | 0,0  | 0,0 | 0,2 | 0,1 | --   |
| III-M                |  |    |  |      |      |     |     |     |      |
| SF                   | 0  | 9  | 0,0  | --   | 0,2  | 0,0 | 0,3 | 0,0 | 0,0  |
| Ti                   | 0  | 10 | 0,1  | --   | 0,0  | 0,2 | 0,0 | 0,2 | 0,0  |
| Am                   | 1  | 10 | 0,0  | --   | 0,0  | 0,0 | 1,2 | 0,0 | 0,0  |
| Ot                   | 8  | 8  | 0,1  | --   | 0,2  | 0,1 | 0,0 | 0,0 | 0,0  |
| SB                   | 0  | 9  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 0,0 | 0,0  |
| Te                   | 0  | 9  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 1,0 | 0,0  |
| II                   |  |    |  |      |      |     |     |     |      |
| JP                   | No   | No | 0,0  | 0,0  | 0,0  | 0,0 | 0,1 | 0,2 | 0,2  |
| Be                   | Yes  | No | 0,4  | 0,1  | 0,4  | 0,2 | 0,3 | 1,5 | 0,4  |
| Ro                   | Yes  | No | 0,0  | 0,0  | 0,1  | 0,0 | 0,0 | 1,0 | 0,1  |
| Ji                   | No   | No | 0,0  | 0,0  | 0,2  | 0,0 | 0,3 | 1,2 | 1,1  |
| Gr*                  | No   | No | 0,1  | 1,4  | 0,2  | 0,4 | 1,3 | 2,4 | 0,3  |
| Ch*                  | Yes  | No | 0,1  | 0,4  | 0,2  | 0,4 | 0,3 | 1,2 | 0,3  |

\*Subjects with facts mastery on Block 1 but not Block 2.

Table I.4 contains the data for those subjects from groups III-H and III-M who demonstrated mastery of both the addition facts and an addition

algorithm. Since both the children who demonstrated knowledge of an addition algorithm and the children who failed to do so were included in groups III-H and III-M, it seems reasonable to assume that the subjects who did evidence mastery of an algorithm had probably not mastered it long before the time of testing. Hence, the five subjects of group III-H and the one subject of group III-M are taken as representative of children who have recently mastered an addition algorithm.

TABLE I.4

## SUBJECTS WHO MASTERED BOTH FACTS AND AN ADDITION ALGORITHM

| Subjects<br>by Group | Numbers of Incorrect Items for Each Variable |    |  |      |      |     |     |     |      |
|----------------------|--|----|--|------|------|-----|-----|-----|------|
|                      | Two-Digit<br>Algorithms                      |    | Tape Block/Time Interval in Seconds<br>(easy items, difficult items) |      |      |     |     |     |      |
|                      | ANC  | AC | 1/15   | 2/15 | 3/10 | 4/8 | 5/6 | 6/4 | 7/10 |
| III-H                |  |    |  |      |      |     |     |     |      |
| BE                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,0 | 0,0 | 0,0  |
| SU                   | 2  | 1  | 0,0  | 0,0  | 0,2  | 0,0 | 0,0 | 0,0 | 0,0  |
| E                    | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,1 | 0,0 | 0,0 | 0,0  |
| JL                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,1 | 0,1 | --   |
| DS                   | 0  | 0  | 0,0  | 0,0  | 0,0  | 0,0 | 0,0 | 0,0 | --   |
| III-M                |  |    |  |      |      |     |     |     |      |
| B                    | 0  | 0  | 0,0  | --   | 0,0  | 0,0 | 0,0 | 0,0 | 0,0  |

The subjects who did not know an algorithm made few errors for the 10- and 8-second interval blocks, with subjects frequently making better scores for the 8-second interval than they did for the 10-second interval. The subjects who did know an algorithm (see Table I.4) missed very few items at any rate of speed, with only one subject missing any item at the 6- and 4-second intervals.

There appeared to be an interaction between the easy and difficult items that resulted in performance generally higher than might occur if all of the items had been either easy or difficult. It was common to observe a child who had not finished "counting up" before the next item

was read; the child would then do the next one if it were easy, and go back to finish or recount the difficult item. The greater speed at which subsequent items were introduced seemed to result primarily in greater concentration on the counting task. Although it was apparent that some of the children were responding immediately with memorized responses, it was not felt that the test separated these children from those who simply increased their rate of counting for the combinations they had not memorized.

### Conclusions About Tape 1

A major objective of an adequate facts speed test is that it separates those subjects who are responding from memory from those who must count to find the solution. It was decided that the test provided by Tape A was inadequate in this regard. It was further judged that the tape was too long for these subjects. It was decided that the data from blocks 1, 2, 3, and 7 indicated that 10-second intervals told as much about mastery of the facts (in terms of the ability to produce sums by counting, if necessary) as did the 15-second intervals.

### Design of Modified Instruments

In light of the above information two additional tape scripts were written. These were the same as Tape 1 with the following modifications:

1. The number of blocks of 10 items was reduced from 7 to 5.
2. The initial warm-up block was reduced to 5 items, 3 easy and 2 difficult, read at 8-second intervals instead of 10.
3. The time intervals for the remaining 4 blocks were 8 seconds, 5 seconds, 3 seconds, and 12 seconds, in that order. The last block was intended for use as a measure of facts mastery.

In addition to the above modifications, Tape B was modified so that in each block of 10 items all five of the easy items were read first. The order of the items for Tape A was the same as for Tape 1. Both Tapes A and B were then recorded by the same reader as Tape 1. A response booklet for use with Tapes A and B was also designed. A copy of this booklet is included in Appendix A. The response booklet contains a final section consisting of 8 two-digit addition items, four of which require regrouping and four of which do not.

In addition to developing Tapes A and B on the basis of information received from the pilot of Tape 1, three written test booklets were also designed. The test booklets all contain the same items as Tapes A and B, written in vertical form with a box in which the student will be asked to write his response.

He is to be further instructed to write only in the box. The order of items is different for the three booklets. The first block of 5 items is identical for all booklets and is the same as the first block of Tape A. The second, third, and fourth blocks of the three booklets consist of the blocks of Tape A, with the order of the blocks changed for the three test booklets. The last block consists of the same items as block 5 of Tape A, but the order of the 10 items within the block is changed for the 3 booklets. The new orders for the last block are random under the constraint that no more than 2 easy or 2 difficult items occur in succession. A final section of 8 items is included in each of the test booklets consisting of 4 two-digit addition items that require regrouping and 4 that do not. The order of the items is random, subject to the constraint that no more than 2 items of either type occur consecutively. The order of these items is different for each of the 3 test booklets. Copies of the test booklets are included in Appendix A. These 3 forms, like Tapes A and B, have not as of this writing been administered to children.

The obvious next step is the piloting of both the tapes and the booklets. It is anticipated that a selection of one of these forms could then be made, and reliability estimates obtained for the selected test.

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Appendix A

TAPESCRIPTS AND PROBLEM SHEETS  
FOR THE ORAL SPEED TESTS



TAPE 1 SCRIPTSPECIFICATIONS:

Narrator reading - 15, 10, 8, 6, 4, 10 second intervals.

SCRIPT:

"Today we are going to see how quickly and accurately you can write sums. I will read a problem such as, 2 plus 3, and you are to write the sum, 5, on the line I tell you to. We will do one set of 10 items for practice."

"Turn your booklet so that the number 1 is at the top of the page."

(15 second pause)

"Problem a, 2 plus 3." (Pause about 5 seconds.) "Since 2 plus 3 is 5 you should write "5" on line a. Do not write, "2 plus 3", write only the 5."

(15 second pause)

"Problem b, 1 plus 7." (Pause about 10 seconds.) "1 plus 7 is 8. Did you write 8 on line b? If not, do that right now. Do not write 1 plus 7; write the sum, 8, only."

"We will pause for just a minute to be sure that you understand."

beep - 5 second pause

The administrators will check to be sure that 5 and 8 are written on lines a and b respectively.

"We will now continue with Problem c through Problem j without stopping. For each of the problems you write the sum on the line of the problem. Work as quickly as you can without making an error. Write only the sum, do not write the problem. If I read a problem so quickly you miss it, skip that line and wait for the next problem."

(About 5 second pause.)

"Ready, begin."

Problem c,  $6 + 6$

(15 second pause)

Problem d,  $2 + 9$

(15 second pause)

Problem e,  $4 + 3$

(15 second pause)

Problem f,  $4 + 9$

(15 second pause)

Problem g,  $3 + 8$

(15 second pause)

Problem h,  $5 + 1$

(15 second pause)

Problem i,  $2 + 8$

(15 second pause)

Problem j,  $7 + 2$

(15 second pause)

"Now that was easy wasn't it? It's all right if you missed a few problems, because the purpose of this exercise is only to see how fast you can write the sums asked for."

(beep - followed by 5 second pause)

"Now turn to the next page so you see the page with the number 2 at the top. (15 second pause.) Remember, do not write the problem, only the sum. Ready, begin."

Problem a,  $1 + 9$

(15 second pause)

Problem b,  $7 + 7$

(15 second pause)

Problem c,  $1 + 4$

(15 second pause)

Problem d,  $7 + 6$

(15 second pause)

Problem e,  $4 + 5$

(15 second pause)

Problem f,  $3 + 6$

(15 second pause)

Problem g,  $5 + 8$

(15 second pause)

Problem h,  $6 + 1$

(15 second pause)

Problem i,  $9 + 8$

(15 second pause)

Problem j,  $5 + 3$

(15 second pause)

"Turn to the next page with the number 3 at the top. (15 second pause.)

The problems for this page, and the following pages, will be read a little faster. If you miss hearing a problem just skip it and wait for the next problem. Ready, begin." (Read at 10 second intervals.)

Problem a,  $8 + 2$

(10 second pause)

Problem b,  $1 + 6$

(10 second pause)

Problem c,  $6 + 8$

(10 second pause)

Problem d,  $9 + 6$

(10 second pause)

Problem e,  $5 + 2$

(10 second pause)

Problem f,  $1 + 5$

(10 second pause)

Problem g,  $3 + 9$

(10 second pause)

Problem h,  $5 + 4$

(10 second pause)

Problem i,  $3 + 3$

(10 second pause)

Problem j,  $7 + 4$

(10 second pause)

"Turn to the next page with the number 4 at the top. (15 second pause.)

These problems will be read a little faster. If you miss hearing a problem just skip it and wait for the next problem. Ready, begin." (Read at 8 second intervals.)

Problem a,  $8 + 4$

(8 second pause)

Problem b,  $2 + 5$

(8 second pause)

Problem c,  $2 + 7$

(8 second pause)

Problem d,  $3 + 7$

(8 second pause)

Problem e,  $9 + 2$

(8 second pause)

Problem f,  $1 + 2$

(8 second pause)

Problem g,  $4 + 4$

(8 second pause)

Problem h,  $5 + 9$

(8 second pause)

Problem i,  $8 + 1$

(8 second pause)

Problem j,  $6 + 7$

(8 second pause)

"Turn to the next page with the number 5 at the top. (15 second pause.)

These problems will be read a little faster. Remember, if you miss hearing a problem, skip it and wait for the next problem. Ready, begin." (Read at 6 second intervals.)

Problem a,  $9 + 5$

(6 second pause)

Problem b,  $4 + 1$

(6 second pause)

Problem c,  $8 + 6$

(6 second pause)

Problem d,  $3 + 2$

(6 second pause)

Problem e,  $7 + 1$

(6 second pause)

Problem f,  $5 + 5$

(6 second pause)

Problem g,  $2 + 4$

(6 second pause)

Problem h,  $9 + 7$

(6 second pause)

Problem i,  $8 + 5$

(6 second pause)

Problem j,  $6 + 3$

(6 second pause)

"Turn to the next page with the number 6 at the top. (15 second pause.)

If you miss hearing a problem just skip it and wait for the next problem.

Ready, begin." (Read at 4 second intervals.)

Problem a,  $2 + 6$

(4 second pause)

Problem b,  $4 + 7$

(4 second pause)

Problem c,  $1 + 8$

(4 second pause)

Problem d,  $3 + 1$

(4 second pause)

Problem e,  $9 + 4$

(4 second pause)

Problem f,  $5 + 6$

(4 second pause)

Problem g,  $3 + 4$

(4 second pause)

Problem h,  $9 + 9$

(4 second pause)

Problem i,  $4 + 6$

(4 second pause)

Problem j,  $4 + 2$

(4 second pause)

"Turn to the next page with the number 7 at the top. (15 second pause.)

These will be read much more slowly. Remember, if you miss hearing a problem just skip it and wait for the next problem. Ready, begin." (Read at 10 second intervals.)

Problem a,  $9 + 1$

(10 second pause)

Problem b,  $1 + 3$

(10 second pause)

Problem c,  $6 + 2$

(10 second pause)

Problem d,  $7 + 9$

(10 second pause)

Problem e,  $8 + 8$

(10 second pause)

Problem f,  $3 + 5$

(10 second pause)

Problem g,  $7 + 8$

(10 second pause)

Problem h,  $2 + 1$

(10 second pause)

Problem i,  $4 + 8$

(10 second pause)

Problem j,  $2 + 2$

(10 second pause)

"Now, close your booklet. Thank you very much for your help."



(SAMPLE PAGE FROM  
RESPONSE BOOKLET FOR TAPE 1)

a. \_\_\_\_\_

b. \_\_\_\_\_

c. \_\_\_\_\_

d. \_\_\_\_\_

e. \_\_\_\_\_

f. \_\_\_\_\_

g. \_\_\_\_\_

h. \_\_\_\_\_

i. \_\_\_\_\_

j. \_\_\_\_\_

name

school

teacher

Find the following sums.

25

$$\begin{array}{r} 48 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 89 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 76 \\ + 14 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 78 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ + 17 \\ \hline \end{array}$$

$$\begin{array}{r} 21 \\ + 52 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 25 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 44 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ + 78 \\ \hline \end{array}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 20 \\ \hline \end{array}$$

$$\begin{array}{r} 19 \\ + 19 \\ \hline \end{array}$$

$$\begin{array}{r} 63 \\ + 22 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ + 39 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \\ + 30 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ + 6 \\ + 48 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ + 25 \\ \hline \end{array}$$

TAPES A AND BSPECIFICATIONS:

Narrator reading at 8, 5, 3, 12 second intervals following warmup  
at 8 second intervals.

SCRIPT:

"Today we are going to see how quickly and accurately you can write sums. I will read a problem such as, 2 plus 3, and you are to write the sum 5 in the box I tell you to. Do not write 2 plus 3, write only the 5. We will do some problems for practice."

"Look at your booklet. Make sure that the number 1 is at the top of the page."

(8 second pause)

"Problem a, 2 plus 3." Remember, write only the sum. (Pause about 5 seconds)  
"Since 2 plus 3 is 5, you should write "5" in box a. Do not write, "2 plus 3", write only the 5."

(6 second pause)

"Problem b, 1 plus 7." (Pause about 8 seconds) "1 plus 7 is 8. Did you write 8 in box b? If not, do that right now. Do not write 1 plus 7; write the sum, 8, only."

"We will pause for just a minute to be sure that you understand."

beep - 5 second pause

The administrators will check to be sure that 5 and 8 are written in boxes a and b respectively.

"We will now continue with Problem c through Problem e without stopping. For each of the problems you write the sum in the box for that problem. Work as quickly as you can without making an error. Write only the sum, do not write the problem. If I read a problem so quickly that you miss it, skip the box and wait for the next problem."

(About 5 second pause.)

"Ready, begin."

Problem c,  $6 + 6$

(8 second pause)

Problem d,  $2 + 9$

(8 second pause)

Problem e,  $4 + 3$

(8 second pause)

"Stop and put your pencil down. Now that was easy wasn't it? It's all right if you missed a few problems, the purpose of this tape is only to see how fast you can write the sums asked for."

(beep - followed by 5 second pause.)

"Now turn to the next page.

(8 second pause.) Remember, do not write the problem, only the sum.

Ready, begin."

(Tape A)

Problem a,  $1 + 9$

(8 second pause)

Problem b,  $7 + 7$

(8 second pause)

Problem c,  $1 + 4$

(8 second pause)

Problem d,  $7 + 6$

(8 second pause)

Problem e,  $4 + 5$

(8 second pause)

(Tape B)

Problem a,  $1 + 4$

(8 second pause)

Problem b,  $4 + 5$

(8 second pause)

Problem c,  $3 + 6$

(8 second pause)

Problem d,  $6 + 1$

(8 second pause)

Problem e,  $5 + 3$

(8 second pause)

## (Tape A)

Problem f,  $3 + 6$ 

(8 second pause)

Problem g,  $5 + 8$ 

(8 second pause)

Problem h,  $6 + 1$ 

(8 second pause)

Problem i,  $9 + 8$ 

(8 second pause)

Problem j,  $5 + 3$ 

(8 second pause)

## (Tape B)

Problem f,  $1 + 9$ 

(8 second pause)

Problem g,  $7 + 7$ 

(8 second pause)

Problem h,  $5 + 8$ 

(8 second pause)

Problem i,  $7 + 6$ 

(8 second pause)

Problem j,  $9 + 8$ 

(8 second pause)

"Stop, put your pencil down and turn to the next page.

(8 second pause) The problems for this page, and the following pages, will be read a little faster. If you miss hearing a problem just skip that box and wait for the next problem. Ready, begin." (Read at 5 second intervals)

## (Tape A)

Problem a,  $8 + 2$ 

(5 second pause)

Problem b,  $1 + 6$ 

(5 second pause)

Problem c,  $6 + 8$ 

(5 second pause)

Problem d,  $9 + 6$ 

(5 second pause)

Problem e,  $5 + 2$ 

(5 second pause)

Problem f,  $1 + 5$ 

(5 second pause)

## (Tape B)

Problem a,  $1 + 6$ 

(5 second pause)

Problem b,  $5 + 2$ 

(5 second pause)

Problem c,  $1 + 5$ 

(5 second pause)

Problem d,  $5 + 4$ 

(5 second pause)

Problem e,  $3 + 3$ 

(5 second pause)

Problem f,  $8 + 2$ 

(5 second pause)

(Tape A)

Problem g,  $3 + 9$ 

(5 second pause)

Problem h,  $5 + 4$ 

(5 second pause)

Problem i,  $3 + 3$ 

(5 second pause)

Problem j,  $7 + 4$ 

(5 second pause)

(Tape B)

Problem g,  $6 + 8$ 

(5 second pause)

Problem h,  $9 + 6$ 

(5 second pause)

Problem i,  $3 + 9$ 

(5 second pause)

Problem j,  $7 + 4$ 

(5 second pause)

"Stop, put your pencil down and turn to the next page.

(8 second pause) These problems will be read a little faster. Remember if you miss hearing a problem just skip that box and wait for the next problem. Ready, begin."

(Read at 3 second intervals).

(Tape A)

Problem a,  $8 + 4$ 

(3 second pause)

Problem b,  $2 + 5$ 

(3 second pause)

Problem c,  $2 + 7$ 

(3 second pause)

Problem d,  $3 + 7$ 

(3 second pause)

Problem e,  $9 + 2$ 

(3 second pause)

Problem f,  $1 + 2$ 

(3 second pause)

(Tape B)

Problem a,  $2 + 5$ 

(3 second pause)

Problem b,  $1 + 2$ 

(3 second pause)

Problem c,  $2 + 7$ 

(3 second pause)

Problem d,  $4 + 4$ 

(3 second pause)

Problem e,  $8 + 1$ 

(3 second pause)

Problem f,  $3 + 7$ 

(3 second pause)

(Tape A)

Problem g,  $4 + 4$ 

(3 second pause)

Problem h,  $5 + 9$ 

(3 second pause)

Problem i,  $8 + 1$ 

(3 second pause)

Problem j,  $6 + 7$ 

(3 second pause)

(Tape B)

Problem g,  $8 + 4$ 

(3 second pause)

Problem h,  $9 + 2$ 

(3 second pause)

Problem i,  $5 + 9$ 

(3 second pause)

Problem j,  $6 + 7$ 

(3 second pause)

"Stop, put your pencil down and turn to the next page. Let's stop for a minute to relax before we do the rest of the problems. (Beep) (8 second pause) These problems will be read much more slowly. As before, write the answer only. Remember, if you miss hearing a problem, skip it and wait for the next problem. Ready, begin." (Read at 12 second intervals).

(Tape A)

Problem a,  $9 + 5$ 

(12 second pause)

Problem b,  $4 + 1$ 

(12 second pause)

Problem c,  $8 + 6$ 

(12 second pause)

Problem d,  $3 + 2$ 

(12 second pause)

Problem e,  $7 + 1$ 

(12 second pause)

Problem f,  $5 + 5$ 

(12 second pause)

(Tape B)

Problem a,  $4 + 1$ 

(12 second pause)

Problem b,  $3 + 2$ 

(12 second pause)

Problem c,  $7 + 1$ 

(12 second pause)

Problem d,  $2 + 4$ 

(12 second pause)

Problem e,  $6 + 3$ 

(12 second pause)

Problem f,  $9 + 5$ 

(12 second pause)

(Tape A)

(Tape B)

Problem g,  $2 + 4$ 

(12 second pause)

Problem h,  $9 + 7$ 

(12 second pause)

Problem i,  $8 + 5$ 

(12 second pause)

Problem j,  $6 + 3$ 

(12 second pause)

Problem g,  $8 + 6$ 

(12 second pause)

Problem h,  $5 + 5$ 

(12 second pause)

Problem i,  $9 + 7$ 

(12 second pause)

Problem j,  $8 + 5$ 

(12 second pause)

"Turn to the next page. (8 second pause.) Try to do all eight of these problems. You will have plenty of time to try them all. If you cannot do some problem, you may guess at an answer, or you may skip that problem.

Thank you very much for your help."



SAMPLE PAGES FROM RESPONSE BOOKLET  
FOR TAPES A and B

1

name

grade

school

teacher

A.

B.

C.

D.

E.

STOP

\_\_\_\_\_

$$\begin{array}{r} 48 \\ + 37 \\ \hline \square \end{array}$$

$$\begin{array}{r} 63 \\ + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 89 \\ + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 21 \\ + 52 \\ \hline \square \end{array}$$

$$\begin{array}{r} 3 \\ + 44 \\ \hline \square \end{array}$$

$$\begin{array}{r} 33 \\ + 39 \\ \hline \square \end{array}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ + 48 \\ \hline \square \end{array}$$

A

1

name

grade school

teacher

RESPONSE BOOKLET  
FOR TEST A

A. 
$$\begin{array}{r} 2 \\ + 3 \\ \hline \square \end{array}$$

B. 
$$\begin{array}{r} 1 \\ + 7 \\ \hline \square \end{array}$$

C. 
$$\begin{array}{r} 6 \\ + 6 \\ \hline \square \end{array}$$

D. 
$$\begin{array}{r} 2 \\ + 9 \\ \hline \square \end{array}$$

E. 
$$\begin{array}{r} 4 \\ + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 2 \\ \text{F.} \quad + 5 \\ \hline \square \end{array}$$

$$\begin{array}{r} 7 \\ \text{G.} \quad + 8 \\ \hline \square \end{array}$$

$$\begin{array}{r} 9 \\ \text{H.} \quad + 1 \\ \hline \square \end{array}$$

$$\begin{array}{r} 1 \\ \text{I.} \quad + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 8 \\ \text{J.} \quad + 8 \\ \hline \square \end{array}$$

$$\begin{array}{r} 2 \\ \text{A.} \quad + 1 \\ \hline \square \end{array}$$

$$\begin{array}{r} 7 \\ \text{B.} \quad + 9 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ \text{C.} \quad + 2 \\ \hline \square \end{array}$$

$$\begin{array}{r} 3 \\ \text{D.} \quad + 5 \\ \hline \square \end{array}$$

$$\begin{array}{r} 4 \\ \text{E.} \quad + 8 \\ \hline \square \end{array}$$

GO ON →

3

$$\begin{array}{r} 6 \\ A. + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 1 \\ B. + 7 \\ \hline \square \end{array}$$

$$\begin{array}{r} 5 \\ C. + 7 \\ \hline \square \end{array}$$

$$\begin{array}{r} 7 \\ D. + 9 \\ \hline \square \end{array}$$

$$\begin{array}{r} 2 \\ E. + 1 \\ \hline \square \end{array}$$

$$\begin{array}{r} 9 \\ F. + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 2 \\ G. + 6 \\ \hline \square \end{array}$$

$$\begin{array}{r} 9 \\ H. + 9 \\ \hline \square \end{array}$$

$$\begin{array}{r} 5 \\ I. + 6 \\ \hline \square \end{array}$$

$$\begin{array}{r} 4 \\ J. + 2 \\ \hline \square \end{array}$$

$$\begin{array}{r} 8 \\ + 7 \\ \hline \square \end{array}$$

F.

$$\begin{array}{r} 5 \\ + 1 \\ \hline \square \end{array}$$

G.

$$\begin{array}{r} 2 \\ + 7 \\ \hline \square \end{array}$$

H.

$$\begin{array}{r} 8 \\ + 9 \\ \hline \square \end{array}$$

I.

$$\begin{array}{r} 2 \\ + 5 \\ \hline \square \end{array}$$

J.

$$\begin{array}{r} 4 \\ + 5 \\ \hline \square \end{array}$$

A.

$$\begin{array}{r} 6 \\ + 5 \\ \hline \square \end{array}$$

B.

$$\begin{array}{r} 7 \\ + 3 \\ \hline \square \end{array}$$

C.

$$\begin{array}{r} 1 \\ + 4 \\ \hline \square \end{array}$$

D.

$$\begin{array}{r} 7 \\ + 5 \\ \hline \square \end{array}$$

E.

GO ON →

$$\begin{array}{r} 6 \\ \text{F. } + 1 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 8 \\ \text{G. } + 3 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{H. } + 6 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 5 \\ \text{I. } + 3 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 9 \\ \text{J. } + 3 \overline{\phantom{00}} \end{array}$$

4

$$\begin{array}{r} 4 \\ \text{A. } + 6 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{B. } + 1 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{C. } + 7 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{D. } + 4 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 1 \\ \text{E. } + 8 \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ + 44 \\ \hline \square \end{array}$$

$$\begin{array}{r} 89 \\ + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 21 \\ + 52 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ + 48 \\ \hline \square \end{array}$$

$$\begin{array}{r} 33 \\ + 39 \\ \hline \square \end{array}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline \square \end{array}$$

$$\begin{array}{r} 48 \\ + 37 \\ \hline \square \end{array}$$

$$\begin{array}{r} 63 \\ + 3 \\ \hline \square \end{array}$$

B

1

name

grade school

teacher

RESPONSE BOOKLET

FOR TEST B

$$\begin{array}{r} 2 \\ A. \quad + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 1 \\ B. \quad + 7 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ C. \quad + 6 \\ \hline \square \end{array}$$

$$\begin{array}{r} 2 \\ D. \quad + 9 \\ \hline \square \end{array}$$

$$\begin{array}{r} 4 \\ E. \quad + 3 \\ \hline \square \end{array}$$



$$\begin{array}{r} 9 \\ \text{F.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{G.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 5 \\ \text{H.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{I.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{J.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{A.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{B.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 9 \\ \text{C.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 1 \\ \text{D.} + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 2 \\ \text{E.} + \overline{\phantom{00}} \end{array}$$

GO ON →

3

$$\begin{array}{r} 4 \\ A. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 3 \\ B. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 6 \\ C. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 7 \\ D. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 2 \\ E. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 8 \\ F. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 1 \\ G. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 9 \\ H. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 4 \\ I. + \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 3 \\ J. + \end{array} \boxed{\phantom{00}}$$

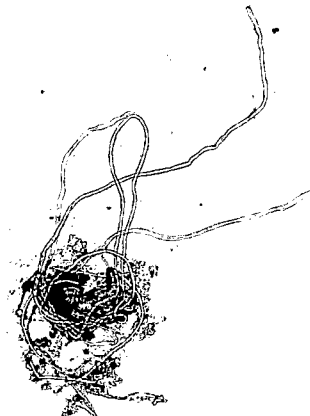
$$\begin{array}{r} 8 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + \square \\ \hline \end{array}$$



STOP

4

$$\begin{array}{r} 4 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + \square \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ F. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 5 \\ G. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 9 \\ H. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 8 \\ I. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 3 \\ J. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 7 \\ A. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 6 \\ B. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 7 \\ C. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 1 \\ D. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 4 \\ E. + \overline{\overline{\square}} \end{array}$$

$$\begin{array}{r} 63 \\ + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 21 \\ + 52 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ + 48 \\ \hline \square \end{array}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline \square \end{array}$$

$$\begin{array}{r} 48 \\ + 37 \\ \hline \square \end{array}$$

$$\begin{array}{r} 89 \\ + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 3 \\ + 44 \\ \hline \square \end{array}$$

$$\begin{array}{r} 33 \\ + 39 \\ \hline \square \end{array}$$

C

I

name

grade

school

teacher

RESPONSE BOOKLET  
FOR TAPE C

$$\begin{array}{r} 2 \\ A. + 3 \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 1 \\ B. + 7 \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 6 \\ C. + 6 \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 2 \\ D. + 9 \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 4 \\ E. + 3 \end{array} \boxed{\phantom{00}}$$

$$\begin{array}{r} 4 \\ + 4 \\ \hline \end{array} \square$$

F.

$$\begin{array}{r} 6 \\ + 9 \\ \hline \end{array} \square$$

G.

$$\begin{array}{r} 5 \\ + 7 \\ \hline \end{array} \square$$

H.

$$\begin{array}{r} 1 \\ + 7 \\ \hline \end{array} \square$$

I.

$$\begin{array}{r} 6 \\ + 4 \\ \hline \end{array} \square$$

J.

$$\begin{array}{r} 2 \\ + 5 \\ \hline \end{array} \square$$

A.

$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array} \square$$

B.

$$\begin{array}{r} 2 \\ + 7 \\ \hline \end{array} \square$$

C.

$$\begin{array}{r} 5 \\ + 1 \\ \hline \end{array} \square$$

D.

$$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array} \square$$

E.

GO ON →

$$\begin{array}{r} 3 \\ \text{A. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{B. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 5 \\ \text{C. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{D. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 9 \\ \text{E. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 2 \\ \text{F. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 1 \\ \text{G. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 9 \\ \text{H. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 3 \\ \text{I. } + \overline{\phantom{00}} \end{array}$$

$$\begin{array}{r} 4 \\ \text{J. } + \overline{\phantom{00}} \end{array}$$



$$\begin{array}{r} 8 \\ + 8 \\ \hline \end{array} \square$$

F.

$$\begin{array}{r} 2 \\ + 1 \\ \hline \end{array} \square$$

G.

$$\begin{array}{r} 6 \\ + 2 \\ \hline \end{array} \square$$

H.

$$\begin{array}{r} 9 \\ + 1 \\ \hline \end{array} \square$$

I.

$$\begin{array}{r} 2 \\ + 2 \\ \hline \end{array} \square$$

J.

$$\begin{array}{r} 7 \\ + 8 \\ \hline \end{array} \square$$

A.

$$\begin{array}{r} 3 \\ + 5 \\ \hline \end{array} \square$$

B.

$$\begin{array}{r} 4 \\ + 8 \\ \hline \end{array} \square$$

C.

$$\begin{array}{r} 7 \\ + 9 \\ \hline \end{array} \square$$

D.

$$\begin{array}{r} 1 \\ + 3 \\ \hline \end{array} \square$$

E.

$$\begin{array}{r} 9 \\ + 3 \\ \hline \square \end{array}$$

F.

$$\begin{array}{r} 3 \\ + 6 \\ \hline \square \end{array}$$

G.

$$\begin{array}{r} 8 \\ + 3 \\ \hline \square \end{array}$$

H.

$$\begin{array}{r} 7 \\ + 5 \\ \hline \square \end{array}$$

I.

$$\begin{array}{r} 6 \\ + 1 \\ \hline \square \end{array}$$

J.

$$\begin{array}{r} 7 \\ + 3 \\ \hline \square \end{array}$$

A.

$$\begin{array}{r} 1 \\ + 4 \\ \hline \square \end{array}$$

B.

$$\begin{array}{r} 4 \\ + 5 \\ \hline \square \end{array}$$

C.

$$\begin{array}{r} 6 \\ + 5 \\ \hline \square \end{array}$$

D.

$$\begin{array}{r} 5 \\ + 3 \\ \hline \square \end{array}$$

E.

$$\begin{array}{r} 21 \\ + 52 \\ \hline \square \end{array}$$

$$\begin{array}{r} 89 \\ + 4 \\ \hline \square \end{array}$$

$$\begin{array}{r} 63 \\ + 3 \\ \hline \square \end{array}$$

$$\begin{array}{r} 3 \\ + 44 \\ \hline \square \end{array}$$

$$\begin{array}{r} 6 \\ + 48 \\ \hline \square \end{array}$$

$$\begin{array}{r} 33 \\ + 39 \\ \hline \square \end{array}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline \square \end{array}$$

$$\begin{array}{r} 48 \\ + 47 \\ \hline \square \end{array}$$

## STUDY II

### AN INVESTIGATION OF THREE DIFFERENT RESPONSE FORMATS FOR A TWO-DIGIT ADDITION AND SUBTRACTION TEST

#### INTRODUCTION

Although the most desirable format for assessing computational skills is a completion format, time and number of children frequently dictate the use of a machine-scorable test. One would like to be confident that the scores obtained on a machine-scorable test do not differ significantly from those scores obtained on a completion test.

The most commonly used machine-scorable test format is multiple choice. The disadvantages of multiple-choice format follow:

1. The child must read through a list of choices until he comes to the correct choice; this may tend to increase the length of the testing period.
2. Many young children are confused when faced with a variety of choices.
3. It is possible for children to guess the correct solution. Thus a score for this child does not necessarily indicate mastery of the behavior being assessed (Anastasi, 1968).
4. Unless the choices include the distractor "none of these," a child has a clue that he has made an error when his solution is not included in the choices.
5. The format frequently prohibits the test's being used for diagnostic purposes. However, it is possible to include the most common type of errors as choice and thus the test could serve as a diagnostic instrument.

In spite of the preceding disadvantages, the obvious advantages of a machine-scorable test (scoring can be done less expensively, more accurately, and in less time than for hand-scored tests) explain the popularity of the multiple-choice format (Bloom et al., 1971). Therefore, it would seem that some attention should be given to investigating other machine-scorable formats which, hopefully, avoid the shortcomings of the multiple-choice format. This study examined one of these alternatives.

#### DETAILS OF THIS STUDY

The purpose of the study was to determine if the response format for a 30-item two-digit addition and subtraction test affected the score. The formats included were as follows:

1. Completion (hand-scored).
2. Multiple choice (machine-scored).
3. Shade the circle (machine-scored).

All three formats are described in detail in the following section.

### Materials

Three response formats were constructed for a 30-item 2-digit addition and subtraction test in which two-thirds of the items involved regrouping; all formats used the same items in the same order. The formats were as follows:

1. Completion. This format consisted of 15 addition problems on one page and 15 subtraction problems on another page. The child was asked to find the sums and differences and to record his response below the item. (Form 1)
2. Multiple choice. This format provided four choices for each item, one of which was "none of these." (Form 2)
3. Shade the circle. The format of the items consisted of an addition or subtraction problem with a three-column grid adjacent to it. The child was expected to compute the solution and then shade the circle in the appropriate column corresponding to the digits in his or her solution.

$$\begin{array}{r} 35 \\ + 21 \\ \hline \end{array}$$

|  |  |  |
|--|--|--|
|  |  |  |
|--|--|--|

|   |   |   |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

In the example, the child would have shaded 6 in the right-hand column and 5 in the middle column. To assist the child in selecting the correct column and the correct digit within the column, space was provided above the columns for the child to record his or her solution; all three practice exercises included this step. (Form 3)

The test forms are included in Appendix B.

### Procedures

Sixty-one second-grade children in an elementary school in Madison, Wisconsin, were randomly assigned to three groups. Each group was administered a form of the test. Forms 2 and 3, the machine-scorable formats, were administered by the authors of the study; Form 1 was administered by a classroom teacher.

### Results

Table II.1 lists the observed means and Hoyt reliabilities for each form.

TABLE II.1  
OBSERVED MEANS AND HOYT RELIABILITIES

| Format           | Numbers | Mean<br>(30 items) | Hoyt<br>Reliabilities |
|------------------|---------|--------------------|-----------------------|
| Completion       | 19      | 17.79              | .83                   |
| Multiple Choice  | 21      | 16.90              | .88                   |
| Shade the Circle | 21      | 18.86              | .94                   |

Two nonindependent comparisons were made: multiple choice versus completion and shade the circle versus completion. The results of the tests are shown in Table II.2.

TABLE II.2  
TESTS ON DIFFERENCES BETWEEN PAIRS OF MEANS

| Comparison                         | df | MS      | F     | Significance |
|------------------------------------|----|---------|-------|--------------|
| Multiple Choice vs.<br>Completion  | 1  | 7.8076  | .2725 | $p < .6037$  |
| Shade the Circle vs.<br>Completion | 1  | 11.3707 | .3969 | $p < .5312$  |

As shown by Table II.2, the differences between the pairs of means can clearly be attributed to chance. As expected, the analysis of variance summary, presented in Table II.3, shows that the overall F ratio was not significant.

TABLE II.3  
ANALYSIS OF VARIANCE SUMMARY

| Source of Variation | df | MS      | F     | Significance |
|---------------------|----|---------|-------|--------------|
| Between Formats     | 2  | 20.0666 | .7005 | $p < .5005$  |
| Error               | 58 | 28.6472 |       |              |

### Discussion

The study showed that for this group of children, the test format did not influence the score on a 30-item 2-digit addition and subtraction test. In particular, the shade-the-circle format, which incorporates all the advantages of a machine-scorable test as well as the diagnostic advantages of a completion test, yielded a mean score not significantly different from

that obtained on the completion test. The authors believe that it is reasonable to assume that similar results--no significant difference between completion and shade-the-circle test formats--would be obtained on any test of computational skills. Thus, if a machine-scorable test is desired and the capability of processing the shade-the-circle test exists, then the shade-the-circle format provides a viable alternative to the multiple-choice format.



### REFERENCES

Anastasi, A. Psychological testing. New York: The Macmillan Company, 1968. Pp. 162-163.

Bloom, S., Hastings, J., & Madaus, F. Handbook on formative and summative evaluation of student learning. New York: McGraw-Hill, 1971. Pp. 253-254.

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Appendix B  
THE ADDITION AND SUBTRACTION TESTS

Name \_\_\_\_\_

Form 1

Teacher \_\_\_\_\_

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Find the Sum

$$\begin{array}{r} 23 \\ + 45 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ + 56 \\ \hline \end{array}$$

$$\begin{array}{r} 25 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ + 50 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ + 58 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \\ + 17 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 52 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 66 \\ + 19 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ + 35 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ + 14 \\ \hline \end{array}$$

$$\begin{array}{r} 75 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ + 24 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ + 44 \\ \hline \end{array}$$

Find the Difference

$$\begin{array}{r} 58 \\ - 23 \\ \hline \end{array}$$

$$\begin{array}{r} 92 \\ - 37 \\ \hline \end{array}$$

$$\begin{array}{r} 85 \\ - 59 \\ \hline \end{array}$$

$$\begin{array}{r} 77 \\ - 52 \\ \hline \end{array}$$

$$\begin{array}{r} 67 \\ - 49 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ - 27 \\ \hline \end{array}$$

$$\begin{array}{r} 52 \\ - 8 \\ \hline \end{array}$$

$$\begin{array}{r} 39 \\ - 16 \\ \hline \end{array}$$

$$\begin{array}{r} 66 \\ - 27 \\ \hline \end{array}$$

$$\begin{array}{r} 93 \\ - 68 \\ \hline \end{array}$$

$$\begin{array}{r} 87 \\ - 77 \\ \hline \end{array}$$

$$\begin{array}{r} 73 \\ - 46 \\ \hline \end{array}$$

$$\begin{array}{r} 70 \\ - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \\ - 42 \\ \hline \end{array}$$

$$\begin{array}{r} 51 \\ - 36 \\ \hline \end{array}$$

Name \_\_\_\_\_

Form 2

Teacher \_\_\_\_\_

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## Example A

|                   |   |               |
|-------------------|---|---------------|
| 25                | 0 | 26            |
| + 3               | 0 | 29            |
| <u>          </u> | ● | 28            |
|                   | 0 | none of these |

## Example B

|                   |   |               |
|-------------------|---|---------------|
| 37                | 0 | 49            |
| - 12              | 0 | 26            |
| <u>          </u> | 0 | 35            |
|                   | 0 | none of these |

Find the Sum

$$\begin{array}{r} 23 \\ + 45 \\ \hline \end{array}$$

0 68  
0 67  
0 69  
0 none of these

$$\begin{array}{r} 37 \\ + 56 \\ \hline \end{array}$$

0 83  
0 93  
0 84  
0 none of these

$$\begin{array}{r} 25 \\ + 49 \\ \hline \end{array}$$

0 65  
0 64  
0 74  
0 none of these

$$\begin{array}{r} 18 \\ + 50 \\ \hline \end{array}$$

0 48  
0 58  
0 68  
0 none of these

$$\begin{array}{r} 29 \\ + 58 \\ \hline \end{array}$$

0 87  
0 77  
0 71  
0 none of these

$$\begin{array}{r} 68 \\ + 17 \\ \hline \end{array}$$

0 75  
0 85  
0 71  
0 none of these

$$\begin{array}{r} 45 \\ + 7 \\ \hline \end{array}$$

0 52  
0 42  
0 53  
0 none of these

$$\begin{array}{r} 52 \\ + 37 \\ \hline \end{array}$$

0 90  
0 85  
0 89  
0 none of these



Find the Sum

$$\begin{array}{r} 66 \\ + 19 \\ \hline \end{array}$$

0 75  
0 85  
0 76  
0 none of these

$$\begin{array}{r} 36 \\ + 35 \\ \hline \end{array}$$

0 71  
0 61  
0 62  
0 none of these

$$\begin{array}{r} 12 \\ + 14 \\ \hline \end{array}$$

0 22  
0 25  
0 26  
0 none of these

$$\begin{array}{r} 75 \\ + 16 \\ \hline \end{array}$$

0 83  
0 93  
0 82  
0 none of these

$$\begin{array}{r} 48 \\ + 6 \\ \hline \end{array}$$

0 42  
0 44  
0 54  
0 none of these

$$\begin{array}{r} 35 \\ + 24 \\ \hline \end{array}$$

0 11  
0 49  
0 59  
0 none of these

$$\begin{array}{r} 37 \\ + 44 \\ \hline \end{array}$$

0 71  
0 73  
0 81  
0 none of these

## Find the Difference

$$\begin{array}{r} 58 \\ - 23 \\ \hline \end{array}$$

0 71  
0 35  
0 36  
0 none of these

$$\begin{array}{r} 92 \\ - 37 \\ \hline \end{array}$$

0 65  
0 55  
0 66  
0 none of these

$$\begin{array}{r} 85 \\ - 59 \\ \hline \end{array}$$

0 26  
0 34  
0 36  
0 none of these

$$\begin{array}{r} 77 \\ - 52 \\ \hline \end{array}$$

0 22  
0 24  
0 25  
0 none of these

$$\begin{array}{r} 67 \\ - 49 \\ \hline \end{array}$$

0 18  
0 28  
0 22  
0 none of these

$$\begin{array}{r} 45 \\ - 27 \\ \hline \end{array}$$

0 22  
0 18  
0 28  
0 none of these

$$\begin{array}{r} 52 \\ - 8 \\ \hline \end{array}$$

0 46  
0 54  
0 44  
0 none of these

$$\begin{array}{r} 39 \\ - 16 \\ \hline \end{array}$$

0 23  
0 45  
0 13  
0 none of these

## Find the Difference

$$\begin{array}{r} 66 \\ - 27 \\ \hline \end{array}$$

☐ 41  
☐ 49  
☐ 39  
☐ none of these

$$\begin{array}{r} 93 \\ - 68 \\ \hline \end{array}$$

☐ 35  
☐ 25  
☐ 36  
☐ none of these

$$\begin{array}{r} 87 \\ - 77 \\ \hline \end{array}$$

☐ 11  
☐ 10  
☐ 1  
☐ none of these

$$\begin{array}{r} 73 \\ - 46 \\ \hline \end{array}$$

☐ 27  
☐ 33  
☐ 37  
☐ none of these

$$\begin{array}{r} 70 \\ - 28 \\ \hline \end{array}$$

☐ 58  
☐ 42  
☐ 52  
☐ none of these

$$\begin{array}{r} 68 \\ - 42 \\ \hline \end{array}$$

☐ 25  
☐ 24  
☐ 26  
☐ none of these

$$\begin{array}{r} 51 \\ - 36 \\ \hline \end{array}$$

☐ 15  
☐ 25  
☐ 26  
☐ none of these

Name \_\_\_\_\_

Form 3

Teacher \_\_\_\_\_

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Example A

$$\begin{array}{r} 25 \\ + 3 \\ \hline 28 \end{array}$$

|   |   |  |
|---|---|--|
| 2 | 5 |  |
|---|---|--|

|   |   |   |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | ● | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | ● |
| 9 | 9 | 9 |

Example B

$$\begin{array}{r} 37 \\ - 12 \\ \hline 25 \end{array}$$

|  |   |   |
|--|---|---|
|  | 3 | 7 |
|--|---|---|

|   |   |   |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | ● | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | ● |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Example C

$$\begin{array}{r} 35 \\ + 21 \\ \hline \end{array}$$

|  |  |  |
|--|--|--|
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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the sum.

$$\begin{array}{r} 23 \\ + 45 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
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| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 37 \\ + 56 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
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| 3 | 3 | 3 |
| 4 | 4 | 4 |
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| 9 | 9 | 9 |

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| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 18 \\ + 50 \\ \hline \end{array}$$

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| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
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| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the sum.

$$\begin{array}{r} 29 \\ + 58 \\ \hline \end{array}$$

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| (0) | (0) | (0) |
| (1) | (1) | (1) |
| (2) | (2) | (2) |
| (3) | (3) | (3) |
| (4) | (4) | (4) |
| (5) | (5) | (5) |
| (6) | (6) | (6) |
| (7) | (7) | (7) |
| (8) | (8) | (8) |
| (9) | (9) | (9) |

$$\begin{array}{r} 68 \\ + 17 \\ \hline \end{array}$$

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|-----|-----|-----|
| (0) | (0) | (0) |
| (1) | (1) | (1) |
| (2) | (2) | (2) |
| (3) | (3) | (3) |
| (4) | (4) | (4) |
| (5) | (5) | (5) |
| (6) | (6) | (6) |
| (7) | (7) | (7) |
| (8) | (8) | (8) |
| (9) | (9) | (9) |

$$\begin{array}{r} 45 \\ + 7 \\ \hline \end{array}$$

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|-----|-----|-----|
| (0) | (0) | (0) |
| (1) | (1) | (1) |
| (2) | (2) | (2) |
| (3) | (3) | (3) |
| (4) | (4) | (4) |
| (5) | (5) | (5) |
| (6) | (6) | (6) |
| (7) | (7) | (7) |
| (8) | (8) | (8) |
| (9) | (9) | (9) |

$$\begin{array}{r} 52 \\ + 37 \\ \hline \end{array}$$

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|-----|-----|-----|
| (0) | (0) | (0) |
| (1) | (1) | (1) |
| (2) | (2) | (2) |
| (3) | (3) | (3) |
| (4) | (4) | (4) |
| (5) | (5) | (5) |
| (6) | (6) | (6) |
| (7) | (7) | (7) |
| (8) | (8) | (8) |
| (9) | (9) | (9) |

Find the sum.

$$\begin{array}{r} 66 \\ + 19 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
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| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 36 \\ + 35 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 12 \\ + 14 \\ \hline \end{array}$$

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|---|---|---|
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| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 75 \\ + 18 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
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| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |



Find the sum.

$$\begin{array}{r} 48 \\ + 6 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 35 \\ + 24 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 37 \\ + 44 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the difference.

$$\begin{array}{r} 58 \\ - 23 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 92 \\ - 37 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 85 \\ - 59 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 77 \\ - 52 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the difference.

$$\begin{array}{r} 67 \\ - 49 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 45 \\ - 27 \\ \hline \end{array}$$

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|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 52 \\ - 8 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 39 \\ - 16 \\ \hline \end{array}$$

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| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the difference.

$$\begin{array}{r} 66 \\ - 27 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 93 \\ - 68 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 87 \\ - 77 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 73 \\ - 46 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
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| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

Find the difference.

$$\begin{array}{r} 70 \\ - 28 \\ \hline \end{array}$$

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| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 68 \\ - 42 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |
| 4 | 4 | 4 |
| 5 | 5 | 5 |
| 6 | 6 | 6 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |
| 9 | 9 | 9 |

$$\begin{array}{r} 51 \\ - 36 \\ \hline \end{array}$$

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| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 2 |
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### STUDY III

## THE EFFECT OF TEST ITEM ORDERING ON THE PERFORMANCE OF CHILDREN OF DIFFERENT TEST ANXIETY LEVELS

### INTRODUCTION

Both school systems and teachers require administration of a great number of examinations of various forms--both teacher-made and commercially produced tests--to assess children's achievement and progress. The value of the results of such tests (in terms of their validity) is dependent upon, among other factors, the conditions under which they are administered and their actual construction. The latter encompasses internal structure--in particular, the idea of sequencing test items according to their difficulty levels. The authors' interests led them to investigate item ordering in mathematics examinations--specifically in computation-type exercises likely to be presented to elementary school pupils.

Much interest and concern exists among educators about providing for individual differences in learning situations. It seems, however, that insufficient attention has been devoted to discovering and implementing means for providing for individual differences in evaluation and testing procedures. While attempts are made to match content and method to pupils' individual needs, too often only one set of criterion measures is applied for evaluation purposes. The present study was designed to take into account one such difference, namely, the amount and "level" of test anxiety in individual school children.

Children evidence anxiety over tests at an early age. Some degrees of anxiety affect individuals' achievement positively, some negatively. Investigations dealing with the effect of test anxiety on a child's performance on arithmetic tests date back several years (Jersild, Goldman, & Loftus, 1940). The existence in fifth- and sixth-grade children of excessive concern about failing a test was noted in their study. Angelino, Dollins, and Mech (1956) found that fears associated with school increased between ages 9 and 12, decreasing thereafter. In that same year, Castaneda, Palermo, and McCandless (1956) studied high and low test anxious fifth-grade children on easy versus difficult components of a complex learning task, noting a statistically significant interaction between anxiety and test complexity.

The intent of the present study was to explore the performance of high, intermediate, and low test anxious fifth-grade children on three different orderings of item difficulty (ascending, descending, and mixed) of an arithmetic computation examination. The null hypotheses of the study are formulated below:

1. No significant difference in the mathematics test scores will be observed between test anxiety levels.
2. No significant difference in the mathematics test scores will be observed between different formats of the examination.
3. No significant interaction between test anxiety level and ordering of examination items will be observed.

## DETAILS OF THIS STUDY

### Subjects

Fifty-three students in two fifth-grade classes in a rural public elementary school in Albany, Wisconsin were selected as the sample for the study.

### Procedure and Design

During the first week of the experiment, Sarason's Test Anxiety Scale for Children (TASC) (Sarason, Davidson, Lighthall, & White, 1958) was administered separately to each class by one of the authors. The class teachers were not present in their classrooms while the scale was administered.

The authors concur with Sarason that the high test anxious child is one who "experiences the test and test-like situations as markedly unpleasant, tinged with more or less vague feelings of uneasiness and bodily tension (Sarason, Lighthall, Davidson, Waite, & Ruebush, 1960, p.11)." The main assumption embedded in his scale is that such anxious reactions form a conscious experience which the child can easily communicate to another person.

In most of his studies, Sarason was concerned with about one-third of the subjects, labeling those students having the top 15 percent of "yes" responses<sup>1</sup> as high test anxious and correspondingly classifying those students having the lowest 15 percent of "yes" responses as low test anxious. Due to the small number of Albany school subjects, however, the authors chose to include all students. An intermediate test anxiety classification was thereby created. Scores on the TASC were divided into

<sup>1</sup>A "yes" response on the scale is interpreted as an admission of an unpleasant experience.

thirds and classified as low (L), intermediate (I), and high (H) test anxiety groups<sup>2</sup> as indicated in Table III.1.

The mathematics computation part of the 1970 edition of the Metropolitan Achievement Test Battery was administered exactly one week after the students had responded to questions on the TASC. Instructions similar to those utilized in a standard testing situation were given orally by the teacher, who was solely responsible for administering the examination.

Preliminary to the actual administering of the Metropolitan test, three forms of the examination--each differing in the ordering of item difficulty--were constructed as indicated in Table III.2. Difficulty of test items was based on their respective p-values.

The forty questions in form A of the examination were ranked in ascending (easy to hard) order of difficulty. The very same items were placed in descending (hard to easy) order of difficulty in form D of the test. Items on form M were arranged in a mixed order of difficulty. To develop this test form, questions were first sorted into 2 groups, the first of which contained 21 test items whose p-values ranged from .60 to .90; the second of which included 19 questions whose p-values were markedly lower (.15 to .55). Questions were then randomly selected, alternating between the 2 groups. The order which resulted became the sequence of form M. The three forms (A, D, and M) of the computation test were then randomly assigned to the subjects within each anxiety level (L, I, H).

Test booklets were marked using a student's assigned row and seat number (e.g., test booklet number 43 corresponded to the child who sat in the third seat of the fourth row). The same seating information was provided by the student on his or her answer sheet for the previously administered TASC. This procedure readily provided the investigators with a complete seating plan of both classes. When the test was administered, the teacher, who was informed about the test labeling technique, was asked to distribute the booklets, thus making sure that each child received the proper, pre-assigned test booklet.

Only scores of those 50 pupils who took both the TASC and the mathematics test were used. Before analyzing the data, 5 cases were discarded in order to use an equal cell 2-way ANOVA computer program, leaving data from 45 students (5 per cell). The 5 cases that were dropped (from 4 cells) were those nearest the individual cell mean.

<sup>2</sup> A similar procedure was used by Walter Szetela in his study (1973), "The Effects of Test Anxiety and Success/Failure on Mathematics Performance in Grade 8."



TABLE III.1

## CLASSIFICATION OF ANXIETY TYPES

| Number of "yes"<br>responses on TASC | Number<br>of Subjects | Classification* |             |
|--------------------------------------|-----------------------|-----------------|-------------|
| 0                                    | 3                     | L               | 16 subjects |
| 1                                    | 3                     | L               |             |
| 2                                    | 2                     | L               |             |
| 3                                    | 0                     | -               |             |
| 4                                    | 0                     | -               |             |
| 5                                    | 1                     | L               |             |
| 6                                    | 2                     | L               |             |
| 7                                    | 5                     | L               |             |
| 8                                    | 5                     | I               | 18 subjects |
| 9                                    | 1                     | I               |             |
| 10                                   | 2                     | I               |             |
| 11                                   | 6                     | I               |             |
| 12                                   | 4                     | I               |             |
| 13                                   | 0                     | -               |             |
| 14                                   | 2                     | H               | 17 subjects |
| 15                                   | 2                     | H               |             |
| 16                                   | 0                     | -               |             |
| 17                                   | 1                     | H               |             |
| 18                                   | 2                     | H               |             |
| 19                                   | 2                     | H               |             |
| 20                                   | 2                     | H               |             |
| 21                                   | 1                     | H               |             |
| 22                                   | 2                     | H               |             |
| 23                                   | 0                     | -               |             |
| 24                                   | 1                     | H               |             |
| 25                                   | 0                     | -               |             |
| 26                                   | 2                     | H               |             |
| 27                                   | 0                     | -               |             |
| 28                                   | 0                     | -               |             |
| 29                                   | 0                     | -               |             |
| 30                                   | 0                     | -               |             |

\* L = low test anxiety  
 I = intermediate test anxiety  
 H = high test anxiety

## FORMAT FOR TEST BOOKLETS

| Test Item Number |        |        |        | Difficulty Index<br>(p-value) |        |        |        |
|------------------|--------|--------|--------|-------------------------------|--------|--------|--------|
| Original         | Form A | Form M | Form D | Original                      | Form A | Form M | Form D |
| 1                | 1      | 13     | 40     | .90                           | .90    | .55    | .15    |
| 2                | 2      | 7      | 39     | .90                           | .90    | .75    | .15    |
| 3                | 3      | 29     | 32     | .85                           | .85    | .40    | .20    |
| 4                | 5      | 20     | 38     | .80                           | .85    | .70    | .25    |
| 5                | 4      | 27     | 37     | .85                           | .80    | .45    | .25    |
| 6                | 6      | 12     | 35     | .80                           | .80    | .70    | .25    |
| 7                | 11     | 30     | 33     | .75                           | .80    | .45    | .25    |
| 8                | 7      | 6      | 28     | .65                           | .75    | .80    | .25    |
| 9                | 9      | 38     | 36     | .75                           | .75    | .25    | .30    |
| 10               | 12     | 8      | 34     | .65                           | .70    | .65    | .30    |
| 11               | 20     | 36     | 26     | .80                           | .70    | .30    | .30    |
| 12               | 8      | 5      | 31     | .70                           | .65    | .85    | .35    |
| 13               | 10     | 33     | 29     | .55                           | .65    | .25    | .40    |
| 14               | 14     | 22     | 30     | .65                           | .65    | .65    | .45    |
| 15               | 15     | 40     | 27     | .65                           | .65    | .15    | .45    |
| 16               | 18     | 18     | 24     | .60                           | .65    | .65    | .45    |
| 17               | 22     | 26     | 23     | .60                           | .65    | .30    | .50    |
| 18               | 16     | 14     | 21     | .65                           | .60    | .65    | .50    |
| 19               | 17     | 31     | 13     | .60                           | .60    | .35    | .55    |
| 20               | 19     | 2      | 25     | .70                           | .60    | .90    | .60    |
| 21               | 25     | 28     | 19     | .50                           | .60    | .25    | .60    |
| 22               | 13     | 10     | 17     | .65                           | .55    | .65    | .60    |
| 23               | 21     | 34     | 16     | .50                           | .50    | .30    | .60    |
| 24               | 23     | 15     | 22     | .45                           | .50    | .65    | .65    |
| 25               | 24     | 37     | 18     | .60                           | .45    | .25    | .65    |
| 26               | 27     | 11     | 15     | .30                           | .45    | .80    | .65    |
| 27               | 30     | 35     | 14     | .45                           | .45    | .25    | .65    |
| 28               | 29     | 9      | 10     | .25                           | .40    | .75    | .65    |
| 29               | 31     | 39     | 8      | .40                           | .35    | .15    | .65    |
| 30               | 26     | 16     | 20     | .45                           | .30    | .60    | .70    |
| 31               | 34     | 23     | 12     | .35                           | .30    | .50    | .70    |
| 32               | 36     | 19     | 9      | .20                           | .30    | .60    | .75    |
| 33               | 28     | 32     | 7      | .25                           | .25    | .20    | .75    |
| 34               | 33     | 1      | 11     | .30                           | .25    | .90    | .80    |
| 35               | 35     | 25     | 6      | .25                           | .25    | .60    | .80    |
| 36               | 37     | 3      | 4      | .30                           | .25    | .85    | .80    |
| 37               | 38     | 24     | 5      | .25                           | .25    | .45    | .85    |
| 38               | 32     | 4      | 3      | .25                           | .20    | .80    | .85    |
| 39               | 39     | 21     | 2      | .15                           | .15    | .50    | .90    |
| 40               | 40     | 17     | 1      | .15                           | .15    | .60    | .90    |

## Results

The grand mean on the mathematics computation test was 24.80 (out of a possible 40). The mean for each treatment cell of the design is provided in Table III.3. The means on the computation test for students in the high, intermediate, and low test anxiety levels are 23.47, 24.00, and 26.93 respectively. The means for students who took the test in which the items were arranged in descending, mixed, and ascending orders of difficulty were 24.20, 26.20, and 24.00 respectively. Table III.4 contains the two-way analysis of variance for scores on the TASC and the computation test.

TABLE III.3

## CELL MEANS

| Mathematics<br>Computation<br>Test Form | Text Anxiety Level |              |       |       |
|---|--------------------|--------------|-------|-------|
|   | High               | Intermediate | Low   | Total |
| D                                       | 24.00              | 24.40        | 24.40 | 24.20 |
| M                                       | 25.00              | 23.20        | 30.40 | 26.20 |
| A                                       | 21.40              | 24.40        | 26.20 | 24.00 |
| Total                                   | 23.47              | 24.00        | 26.93 | 24.80 |

TABLE III.4

## ANALYSIS OF VARIANCE SUMMARY

| Sources        | Sum of<br>Squares | Degrees<br>of Freedom | Mean<br>Squares | F ratio |
|----------------|-------------------|-----------------------|-----------------|---------|
| Anxiety Levels | 104,533           | 2                     | 52,267          | 1.364   |
| Test forms     | 44,400            | 2                     | 22,200          | .579    |
| Interaction    | 95,066            | 4                     | 23,767          | .620    |
| Error          | 1,379,200         | 36                    | 38,311          |         |
| Total          | 1,623,200         | 44                    |                 |         |

The F statistics for the effect due to the level of test anxiety as well as to the ordering of questions on the mathematics computation test were not significant at  $\alpha = .05$  ( $p < .025$ ). The F statistics at  $\alpha = .05$  were not significant for the interaction of the test anxiety levels and the mathematics test item orderings. On the basis of these results, none of the three previously stated null hypotheses can be rejected.

In general, students who took the mixed form of the mathematics test tended to score higher than those who did not. This trend was somewhat constant across test anxiety levels. Low test anxious children seemed to perform better on the mathematics test when compared to their peers. This trend was constant across the three forms of the examination. The highest cell mean was obtained from low test anxious students who took the mixed form of the examination.

### Discussion

Low test anxiety seemed to have a rather facilitating effect on the mathematics test performance of the children. That students who took the mixed form of the examination scored higher than their classmates may be attributable to the possibility that such item difficulty sequences are more intellectually arousing or provoking than the other two forms. It is surprising that there is very little difference between the means obtained from students who took form A and those who took form D of the test. No explanation for this occurrence is offered.

The statistically non-significant results of the study may have been due to the small number of subjects in each cell ( $n = 5$ ). Perhaps with a slightly larger sample, different outcomes and trends would have been observed. Another point of particular concern to the authors relates to classification of students into high, intermediate, and low test anxiety groups. Cut-off point--in particular, the one between low and intermediate test anxious subgroups--were not very distinct. Although 30 students scored below and 20 pupils performed better than the mean on the mathematics computation test, overall results were not strikingly unique (see Figure III.1). The distribution of scores was positively skewed.

Studies of this nature need to be replicated with a larger sample, possibly combined with different cut-off points and more precise instruments.

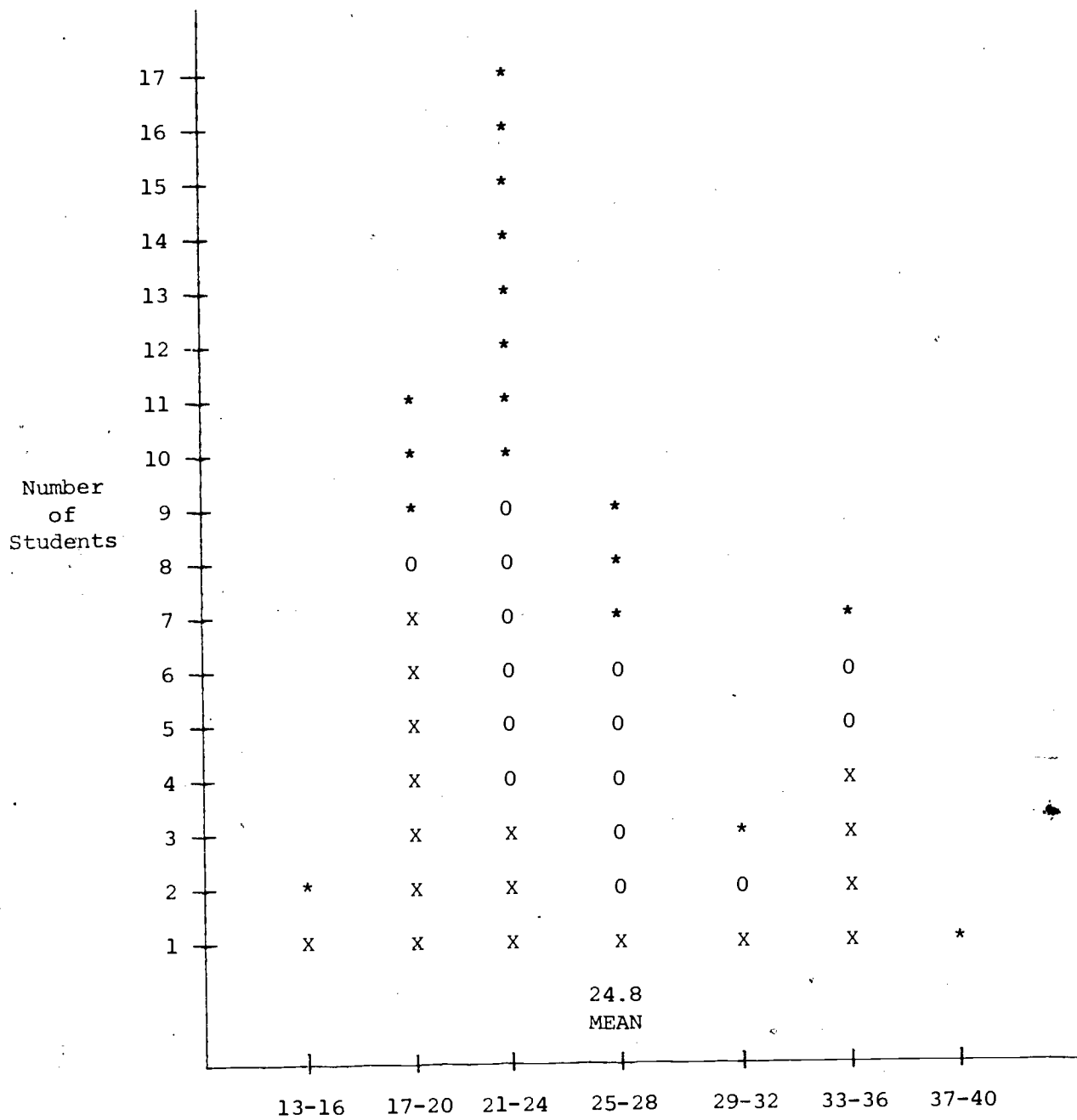


Figure III.1. Mathematics computation test scores.

X = Form D  
O = Form M  
\* = Form A

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## STUDY IV

### A COMPARATIVE STUDY OF THREE REMEDIAL METHODS OF INSTRUCTION FOR TWO-DIGIT ADDITION WITH REGROUPING AT THE SECOND GRADE

#### INTRODUCTION

This investigation deals with the remedial component of mathematics instruction. A variety of factors contribute to differences in understanding of a given mathematical unit, and in the student achievement.

One factor is the use of a method of instruction that does not take into account the student's capabilities or his learning style. In some cases a student learns in a "one track" way and his low achievement is due to a teaching method that does not match his way of learning. As Reissman says,

A child may be slow because he learns in what I have called a "one tract" way; that is, he persists in one line of thought and is not flexible or broad. He does not easily adapt to other frames of reference, such as the teacher's, and consequently he may appear slow and dull (1962, p. 226).

Other students have a learning style based primarily on a confrontation with the immediate environment and, consequently, need to manipulate physical material in order to master a given skill or to grasp a given idea.

According to Schulz,

While the importance of visual and action-based mathematical experiences has been hypothesized on a neurological basis for all children, tactile experiences with objects and events are indispensable to the learner who has no other effective input channel. To form concepts and work mathematical problems, he needs the physical input provided by such manipulative materials as fraction pieces, Dienes blocks, geoboards, puzzles, games, machine calculators, and in fact the entire mathematics laboratory (1972, p. 5).

One reason given for the low achievement of many students is the stress on meaning in the present math programs. The assumption is that, if a student understands what he is doing, the skill will necessarily follow. Anderson (1949) investigated the effect of meaningful instruction versus rote instruction on achievement, retention, and transfer of learning of fourth-grade students. He concluded that the meaning approach is suitable for students with high IQ's, while the rote method is more appropriate for students with low IQ's, with some qualification.

Later investigations indicate that, for many children, understanding may come more appropriately after the acquisition of skills, and that this type of learning can be suitable for children with high IQ's as well as for those with low IQ's. Willoughby pointed to the results of these investigations in the following terms:

In fact it seems distinctly possible that, for many children, understanding may come more appropriately after or during acquisition of the skills and of the ability to verbalize. Although there are many strongly held opinions on this subject, there seems to be little clear evidence regarding which children should learn which mathematics in which ways (1970, p. 264).

The effect of these considerations on instructional methods in general, and remedial instruction in particular, has given rise to several practical questions. Following are three of these questions:

1. Should children receive remedial instruction within the same context under which they were initially instructed?
2. Are there motivating devices available which might make the task of remediation of skills easier?
3. Should meaning and understanding be sacrificed at some point in favor of a rote approach to the instruction of computational skills?

This study attempts to shed some light on the previous three questions. It deals with the remediation of two-digit addition with regrouping in a second-grade setting, and investigates the following three approaches to remedial instruction:

1. The use of a hand-held calculator to facilitate instruction.
2. An approach that uses sub-sums.
3. The traditional carrying approach.

## DETAILS OF THE STUDY

The procedure consisted of three parts: initial testing, instruction, and final testing. A description of each of these parts follows.

### Initial Testing

The purpose of the initial testing was to identify those students who could profit from remedial instruction on two-digit addition with regrouping. The entire second grade (89 students) of the Zachary Lane Elementary School of the Robbinsdale Area Schools in suburban Minneapolis received this testing. All students had received basic instruction in



two-digit addition with regrouping through expanded notation progressing to the traditional carrying approach. The initial test consisted of four parts:

1. Ten teen facts.
2. Five two-digit addition problems with no regrouping.
3. Five two-digit addition problems with regrouping.
4. Five three-digit addition problems with regrouping.

Part 1 of the test was to identify those students who had basic fact problems, part 2 was to give additional information on these facts as well as a basic understanding of two-digit addition. Part 3 was intended to give information on those students who could not perform two-digit addition with regrouping and could profit from the instruction. Part 4 was included in order to provide a safeguard for part 3 and, at the same time, to give initial information about eventual transfer.

Five forms of the test were produced (see Appendix D). Each of these forms consisted of problems randomly generated by a computer. The following ideal score pattern was expected:

| <u>Part 1</u> | <u>Part 2</u> | <u>Part 3</u> | <u>Part 4</u> |
|---------------|---------------|---------------|---------------|
| 10 correct    | 5 correct     | 0 correct     | 0 correct     |

The tests were arranged so that students in each classroom used each of the five forms. The tests were distributed and supervised by the classroom teachers.

The results of the initial testing are given in Table IV.1.

TABLE IV.1  
FREQUENCY DISTRIBUTION ON THE INITIAL TEST

|        |                |    |    |    |   |   |   |    |   |   |   |   |
|--------|----------------|----|----|----|---|---|---|----|---|---|---|---|
| Part 1 | Number correct | 10 | 9  | 8  | 7 | 6 | 5 | 4  | 3 | 2 | 1 | 0 |
|        | Frequency      | 63 | 9  | 8  | 3 | 2 | 2 | 1  | 1 | 0 | 0 | 0 |
| Part 2 | Number correct |    | 5  | 4  | 3 | 2 | 1 | 0  |   |   |   |   |
|        | Frequency      |    | 74 | 11 | 3 | 1 | 0 | 0  |   |   |   |   |
| Part 3 | Number correct |    | 5  | 4  | 3 | 2 | 1 | 0  |   |   |   |   |
|        | Frequency      |    | 44 | 12 | 2 | 0 | 1 | 30 |   |   |   |   |
| Part 4 | Number correct |    | 5  | 4  | 3 | 2 | 1 | 0  |   |   |   |   |
|        | Frequency      |    | 42 | 12 | 1 | 1 | 0 | 33 |   |   |   |   |

Fifteen students whose scores approached the ideal score pattern were chosen; their scores are given in Table IV.2. They were randomly assigned to three instruction groups so that each class was represented in each group.

TABLE IV.2  
SCORE PATTERNS ON INITIAL TEST  
OF CHILDREN IN INSTRUCTION GROUPS

| Student   | Part 1 | Part 2 | Part 3 | Part 4 |
|-----------|--------|--------|--------|--------|
| Group I-1 |        |        |        |        |
| 1         | 10     | 5      | 0      | 0      |
| 2         | 10     | 5      | 0      | 0      |
| 3         | 10     | 4      | 0      | 0      |
| 4         | 10     | 4      | 0      | 0      |
| 5         | 9      | 5      | 0      | 0      |
| Group I-2 |        |        |        |        |
| 1         | 10     | 5      | 0      | 0      |
| 2         | 10     | 5      | 0      | 0      |
| 3         | 9      | 5      | 0      | 0      |
| 4         | 9      | 5      | 0      | 0      |
| 5         | 8      | 5      | 0      | 0      |
| Group I-3 |        |        |        |        |
| 1         | 10     | 5      | 0      | 0      |
| 2         | 10     | 5      | 0      | 0      |
| 3         | 10     | 5      | 0      | 0      |
| 4         | 10     | 5      | 0      | 0      |
| 5         | 10     | 4      | 0      | 0      |

### Instruction

Instruction was carried out by an investigator over two periods, one of 30 minutes and one of 15 minutes, in the following way:

#### First Group: The calculator method

The instruction for this group was built around the use of hand-held calculators. It consisted of three parts:

1. Introduction to the calculator
2. Solving two-digit additions with regrouping using the calculator
3. Solving two-digit additions with regrouping without the calculator, and then using the calculator to check the solution

#### Second Group: The sub-sum method

In this group, sub-sums were used as shown by the following example:

|         |           |          |           |
|---------|-----------|----------|-----------|
| Problem | 27        | Solution | 27        |
|         | <u>35</u> |          | <u>35</u> |
|         |           |          | 12        |
|         |           |          | <u>50</u> |
|         |           |          | 62        |

The instruction consisted of exploring with the students how the two sub-sums were generated and how the solution was obtained.

#### Third Group: The traditional carrying method

The instruction for this group is illustrated by the following example:

|         |           |          |           |
|---------|-----------|----------|-----------|
| Problem | 27        | Solution | 1         |
|         | <u>35</u> |          | 27        |
|         |           |          | <u>35</u> |
|         |           |          | 62        |

The students were told that 7 + 5 has to be considered first. The sum, 12, is then broken into two parts--the 2 is to be placed directly under the 7 and 5 in the ones position, and the 1 is to be placed in the tens column. The tens can then be added, resulting in a sum of 62.

In the first group, interest in the use of the calculators was high. However, when the second phase began (solving problems and checking with the calculator) it was apparent that three children in the group were having trouble with basic number facts. It was also apparent that these students did not want to shift to phase two because of their success with using the calculator during phase one.

The children in the second group expressed some disappointment because they could not use the calculator. They experienced success in using the sub-sum approach. Two students in this group had some trouble with the basic facts, but not as severe as the students in the first group.

The students in the third group were extremely happy solving problems using the traditional carrying method. They worked more problems as a group than either one of the previous two groups. They realized that their success was high and were content with this success. None of the children in this group had basic fact problems. The number of practice problems completed by each child is shown in Table IV.3.

TABLE IV.3  
NUMBER OF PRACTICE PROBLEMS COMPLETED

| Student | Group |     |     |
|---------|-------|-----|-----|
|         | I-1   | I-2 | I-3 |
| 1       | 47    | 57  | 69  |
| 2       | 30    | 98  | 80  |
| 3       | 28    | 36  | 134 |
| 4       | 32    | 14  | 40  |
| 5       | 23    | 16  | 114 |
| Total   | 160   | 221 | 437 |

#### Final Testing

The posttest was administered by the investigator to all of the students at the same time. This posttest took 20 minutes and consisted of two parts:

1. Ten two-digit addition problems with regrouping
2. Five three-digit addition problems with regrouping

The problems were randomly generated by a computer (see Appendix C). The results of the posttesting are given in Table IV.4.

TABLE IV.4  
NUMBER OF CORRECT ANSWERS ON THE POSTTEST

| Student   | Part One | Part Two |
|-----------|----------|----------|
| Group I-1 |          |          |
| 1         | 9        | 1        |
| 2         | 10       | 3        |
| 3         | 3        | 0        |
| 4         | 9        | 0        |
| 5         | 4        | 0        |
| Group I-2 |          |          |
| 1         | 7        | 0        |
| 2         | 10       | 0        |
| 3         | 9        | 0        |
| 4         | 6        | 0        |
| 5         | 10       | 0        |
| Group I-3 |          |          |
| 1         | 10       | 0        |
| 2         | 10       | 5        |
| 3         | 10       | 4        |
| 4         | 10       | 5        |
| 5         | 9        | 5        |

## Results

Table IV.5 shows the mean and standard deviation of the part 1 score for each of the three groups.

TABLE IV.5  
POSTTEST MEANS AND STANDARD DEVIATIONS

|         | Mean | Standard<br>Deviation |
|---------|------|-----------------------|
| Group 1 | 7    | 2.9                   |
| Group 2 | 8.4  | 1.62                  |
| Group 3 | 9.8  | .4                    |

This table shows a marked trend in favor of method 3. It also shows method 2 favored over method 1.

Table IV.6 shows a one-way analysis of variance with a fixed effects model of these scores.

TABLE IV.6  
ANOVA TABLE FOR POSTTEST SCORES

| Source         | DF | SS   | MS  | F                        |
|----------------|----|------|-----|--------------------------|
| Between Groups | 2  | 19.6 | 9.8 | $\frac{9.8}{4.6} = 2.12$ |
| Within Groups  | 12 | 56   | 4.6 |                          |
| Total          | 14 | 75.6 |     |                          |

Since the  $F_{2.12}$  statistics at the .1 level is equal to 2.81, the three methods of instruction do not differ significantly.

It is obvious from Table IV.4 that the third method yielded transfer to the three-digit addition problems with regrouping, while very little transfer was yielded by methods 1 and 2.

## RETENTION STUDY AND DATA

It was decided that a retention test would shed more light on the data in Table IV.4.

This test was identical in number of problems and form to the final test and was administered exactly 11 days after the final test (see Appendix D). It should be noted that a spring vacation period occurred in the interval between the two testings and no instruction took place during this period. The results of the retention testing are listed in Table IV.7.

TABLE IV.7  
NUMBER OF CORRECT ANSWERS ON THE RETENTION TEST

| Student   | Part 1 | Part 2 |
|-----------|--------|--------|
| Group I-1 |        |        |
| 1         | 6      | 0      |
| 2         | 5      | 0      |
| 3         | 3      | 0      |
| 4         | 8      | 5      |
| 5         | 0      | 0      |
| Group I-2 |        |        |
| 1         | 9      | 0      |
| 2         | 10     | 0      |
| 3         | 7      | 0      |
| 4         | 4      | 0      |
| 5         | 10     | 0      |

(continued on next page)

Table IV.7 (Cont.)

| Student   | Part 1 | Part 2 |
|-----------|--------|--------|
| Group I-3 |        |        |
| 1         | 5      | 0      |
| 2         | 10     | 3      |
| 3         | 10     | 4      |
| 4         | 10     | 5      |
| 5         | 9      | 4      |

Results of the Retention Test

Table IV.8 shows the mean and standard deviation of the part 1 score for each of the three groups.

TABLE IV.8

## RETENTION MEANS AND STANDARD DEVIATIONS

|         | Mean | Standard<br>Deviation |
|---------|------|-----------------------|
| Group 1 | 4.4  | 2.73                  |
| Group 2 | 8    | 2.28                  |
| Group 3 | 8.8  | 1.94                  |

Table IV.9 shows a one-way analysis of variance with a fixed effects model of the scores on the retention test.



TABLE IV.9  
ANOVA TABLE FOR RETENTION TEST SCORES

| Source         | DF | SS | MS   | F                          |
|----------------|----|----|------|----------------------------|
| Between Groups | 2  | 55 | 27.5 | $\frac{27.5}{6.75} = 4.07$ |
| Within Groups  | 12 | 81 | 6.75 |                            |

Since an  $F_{2,12}$  at the .05 level of significance is 3.89, the scores of the three groups differ significantly. From the Table IV.8 this difference is due mainly to the low scores of the first group. As for the transfer of learning to the three-digit addition with regrouping, the retention test data are in accord with those of the posttesting.

### Discussion

Table IV.5 shows that the three methods used in the study can provide an increase in achievement in two-digit addition with regrouping.

The use of hand-held calculators did not yield better achievement in the posttest than either methods 2 or 3. A similar result was obtained by Cech (1972) in a study conducted to investigate the effect of the use of desk calculators on attitudes and achievements with low-achieving ninth graders. Moreover, the results of the retention test show a significant difference in favor of methods 2 and 3.

It is quite clear from the data that the traditional carrying method did transfer to three-digit addition with regrouping to a far greater degree than did either of the other two methods of instruction. Concerning the achievement, Table IV.4 shows a marked trend in favor of method 3. However, no conclusion can be drawn from this study about whether children should receive remedial instruction within the same context under which they were initially instructed. One reason for the failure to answer the previous question is that none of the students in Group 3 had troubles with the basic number facts, while three students from the first group and two from the second had such troubles. It is very probable that the background of the students has affected the achievement results.

It is the opinion of the investigators that the trend in favor of the third method could be attributed to the background of the students, to the method itself, or to the fact that the children's initial instruction was with this same method. A replication of the study might help elucidate this point.

The instruction given did not stress any relationship between meanings and computations. A useful study might be undertaken to investigate the effect of rote instruction on subsequent understanding of the unresolved algorithms.

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Appendix C  
ADDITION TESTS

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## RETENTION TEST

Name \_\_\_\_\_

ADD

$$\begin{array}{r} 29 \\ + 32 \\ \hline \end{array}$$

$$\begin{array}{r} 39 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ + 47 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ + 45 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ + 29 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ + 117 \\ \hline \end{array}$$

$$\begin{array}{r} 613 \\ + 219 \\ \hline \end{array}$$

$$\begin{array}{r} 285 \\ + 291 \\ \hline \end{array}$$

$$\begin{array}{r} 164 \\ + 353 \\ \hline \end{array}$$

$$\begin{array}{r} 693 \\ + 158 \\ \hline \end{array}$$

Name \_\_\_\_\_

Section \_\_\_\_\_

ADD

$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ + 65 \\ \hline \end{array}$$

$$\begin{array}{r} 41 \\ + 32 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ + 62 \\ \hline \end{array}$$

$$\begin{array}{r} 53 \\ + 20 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ + 43 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ + 32 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 15 \\ + 65 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ + 48 \\ \hline \end{array}$$

$$\begin{array}{r} 49 \\ + 33 \\ \hline \end{array}$$

$$\begin{array}{r} 347 \\ + 249 \\ \hline \end{array}$$

$$\begin{array}{r} 747 \\ + 238 \\ \hline \end{array}$$

$$\begin{array}{r} 283 \\ + 674 \\ \hline \end{array}$$

$$\begin{array}{r} 455 \\ + 271 \\ \hline \end{array}$$

$$\begin{array}{r} 693 \\ + 158 \\ \hline \end{array}$$

Name \_\_\_\_\_

Section \_\_\_\_\_

ADD

$$\begin{array}{r} 4 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ + 43 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ + 64 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ + 41 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ + 44 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ + 44 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ + 29 \\ \hline \end{array}$$

$$\begin{array}{r} 21 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 39 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 238 \\ + 409 \\ \hline \end{array}$$

$$\begin{array}{r} 469 \\ + 322 \\ \hline \end{array}$$

$$\begin{array}{r} 543 \\ + 296 \\ \hline \end{array}$$

$$\begin{array}{r} 164 \\ + 353 \\ \hline \end{array}$$

$$\begin{array}{r} 397 \\ + 536 \\ \hline \end{array}$$

Name \_\_\_\_\_

Section \_\_\_\_\_

ADD

$$\begin{array}{r} 8 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 62 \\ + 21 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ + 30 \\ \hline \end{array}$$

$$\begin{array}{r} 30 \\ + 68 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \\ + 13 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ + 31 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ + 46 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 16 \\ + 65 \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 479 \\ + 416 \\ \hline \end{array}$$

$$\begin{array}{r} 324 \\ + 358 \\ \hline \end{array}$$

$$\begin{array}{r} 285 \\ + 291 \\ \hline \end{array}$$

$$\begin{array}{r} 322 \\ + 184 \\ \hline \end{array}$$

$$\begin{array}{r} 398 \\ + 94 \\ \hline \end{array}$$



Name \_\_\_\_\_

Section \_\_\_\_\_

ADD

$$\begin{array}{r} 3 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 3 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ + 24 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ + 11 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ + 52 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ + 27 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \\ + 42 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ + 17 \\ \hline \end{array}$$

$$\begin{array}{r} 64 \\ + 16 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ + 45 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ + 43 \\ \hline \end{array}$$

$$\begin{array}{r} 613 \\ + 219 \\ \hline \end{array}$$

$$\begin{array}{r} 317 \\ + 346 \\ \hline \end{array}$$

$$\begin{array}{r} 433 \\ + 376 \\ \hline \end{array}$$

$$\begin{array}{r} 167 \\ + 651 \\ \hline \end{array}$$

$$\begin{array}{r} 788 \\ + 187 \\ \hline \end{array}$$

Name \_\_\_\_\_

Section \_\_\_\_\_

ADD

$$\begin{array}{r} 8 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ + 6 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ + 24 \\ \hline \end{array}$$

$$\begin{array}{r} 61 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ + 46 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ + 74 \\ \hline \end{array}$$

$$\begin{array}{r} 84 \\ + 13 \\ \hline \end{array}$$

$$\begin{array}{r} 49 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ + 28 \\ \hline \end{array}$$

$$\begin{array}{r} 76 \\ + 17 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ + 47 \\ \hline \end{array}$$

$$\begin{array}{r} 219 \\ + 244 \\ \hline \end{array}$$

$$\begin{array}{r} 327 \\ + 117 \\ \hline \end{array}$$

$$\begin{array}{r} 273 \\ + 175 \\ \hline \end{array}$$

$$\begin{array}{r} 642 \\ + 162 \\ \hline \end{array}$$

$$\begin{array}{r} 279 \\ + 459 \\ \hline \end{array}$$

Name \_\_\_\_\_

ADD

$$\begin{array}{r} 14 \\ + 56 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ + 25 \\ \hline \end{array}$$

$$\begin{array}{r} 69 \\ + 24 \\ \hline \end{array}$$

$$\begin{array}{r} 25 \\ + 38 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ + 69 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 39 \\ + 49 \\ \hline \end{array}$$

$$\begin{array}{r} 47 \\ + 45 \\ \hline \end{array}$$

$$\begin{array}{r} 54 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 67 \\ + 27 \\ \hline \end{array}$$

$$\begin{array}{r} 338 \\ + 214 \\ \hline \end{array}$$

$$\begin{array}{r} 467 \\ + 127 \\ \hline \end{array}$$

$$\begin{array}{r} 182 \\ + 672 \\ \hline \end{array}$$

$$\begin{array}{r} 126 \\ + 582 \\ \hline \end{array}$$

$$\begin{array}{r} 475 \\ + 327 \\ \hline \end{array}$$

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